COURSE CONTENT

SUBECT: DESIGN OF MACHINE ELEMENT

UNIT-IV

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ASST. PROF. ME DEPT

Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and ar e subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view :

1. Sunk keys, **2.** Saddle keys, **3.** Tangent keys, **4.** Round keys, and **5.** Splines.

Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyw ay of the hub or boss of the pulley. The sunk keys are of the following types :

1. Rectangular sunk key. A recctangular sunk key is shown in Fig. The usual proportions of this key are :

Width of key, $w = d / 4$; and thi ckness of key, $t = 2w / 3 = d / 6$
where d = Diameter of the shaft or diameter of the hole in the
hub. The key has taper 1 in 100 on th e top side only.

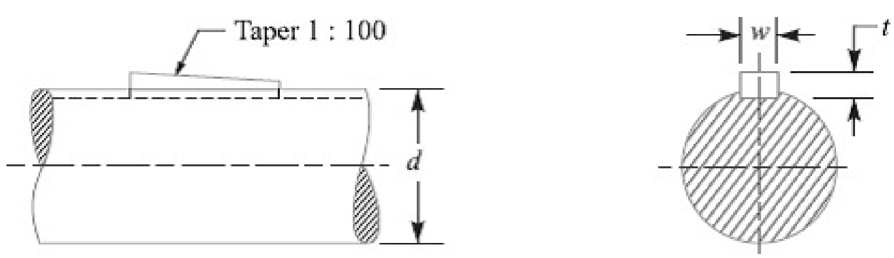


Fig. Sunk Key

2. Square sunk key. The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e. $w = t = d / 4$

3. Parallel sunk key. The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taper less and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. Gib-head key. It is a rectangular sunk key with a head at one end known as **gib head.**

It is usually provided to facilitate the removal of key. A gib head key is shown i n Fig. (A)and its use in shown in Fiig. (b).

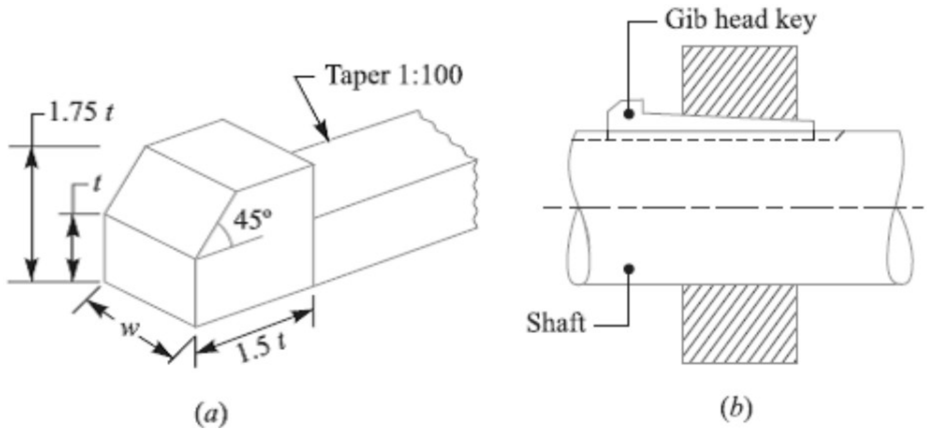


Fig. Gib head key and its use.

The usual proportions of the gib head key are:

Width, w = d / 4 ; and thickness at large end, t = 2w / 3 = d / 6

5. Feather key. A key attached to one member of a pair and which permits relative axial movement is known as **feather key.** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the sh aft or hub, the key being a sliding fit in the key way of the moving piece.

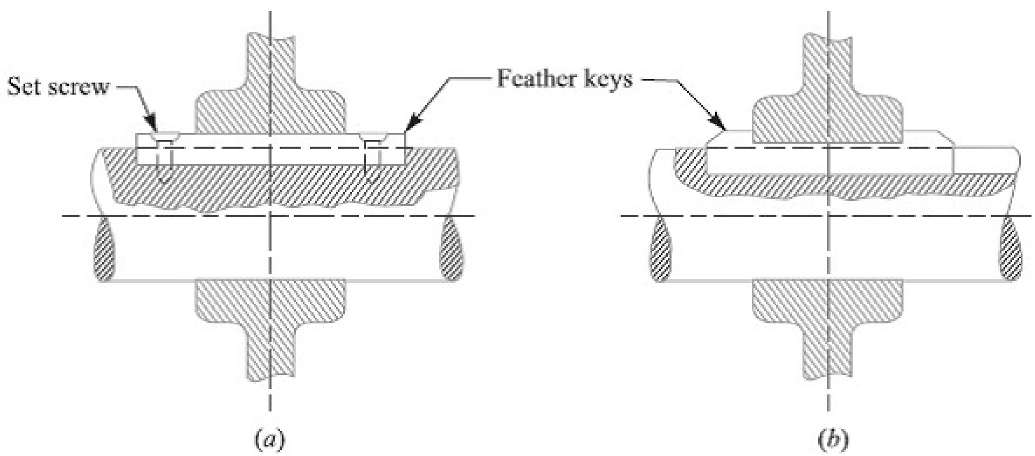


Fig. Feather Keys

6. Woodruff key. The woodr uff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

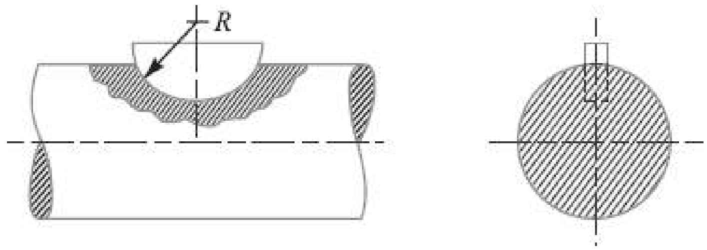


Fig. Woodruff Key

The main advantages of a woodruff key are as follows:

1. It accommodates itself to any taper in the hub or boss of the mating piece.

2. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any ten dency to turn over in its keyway.

The disadvantages are:

1. The depth of the keyway weak ens the shaft.

2. It can not be used as a feather.

Saddle keys

The saddle keys are of the follow ing two types:

1. Flat saddle key, and **2.** Hollow saddle key.

A **flat saddle key** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

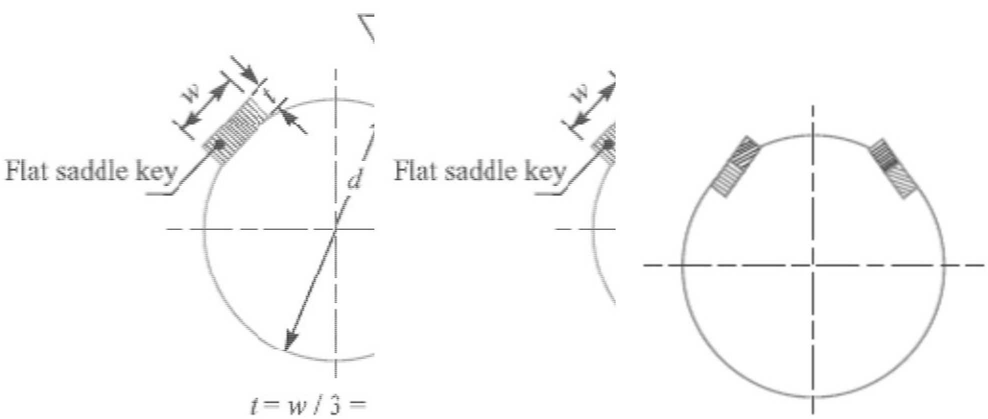


Fig. Flat saddle key and Tangent keys

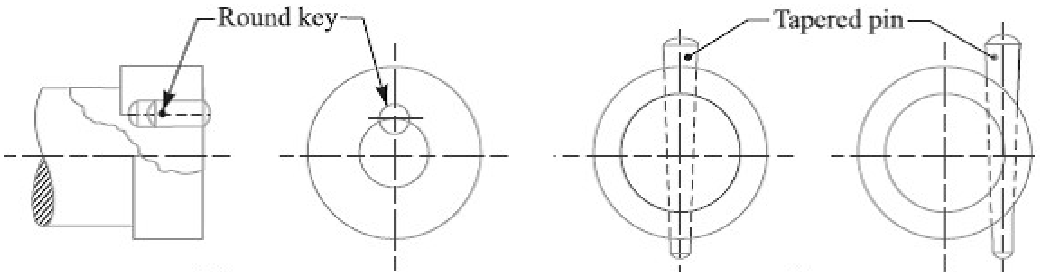
A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

Tangent Keys

The tangent keys are fitted in p air at right angles as shown in Fig. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

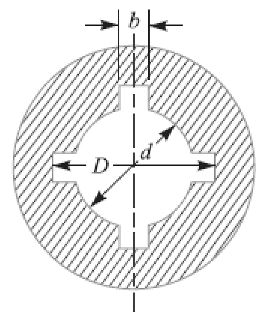
Round Keys

The round keys, as shown in Fig . (a) are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways m ay be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.



Splines

Sometimes, keys are made integgral with the shaft which fits in the keyways broached in the hub. Such shafts are known as **splined shafts** as shown in Fig . These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.



$D = 1.25\ d$ and $b = 0.25\ D$

Stresses in Keys:

Forces acting on a Sunk Key

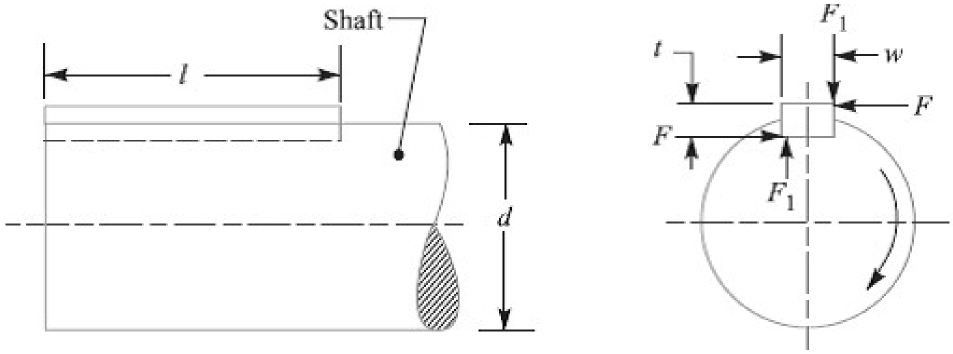
When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key:

1. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straigh t key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig.

In designing a key, forces due to fit of the key are neglected and it is assu med that the distribution of forces along the leength of key is uniform.



Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig.

Let T = Torque transmitted by the shaft,

F = Tangential force acti ng at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

τ **and σ**c = Shear and cru shing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft,, the key may fail due to shearing or crushing. Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

F = Area resisting shearing × Shear stress = l × w ×

τ Therefore, Torque transmitted byy the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad \text{...(i)}$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

F = Area resisting crushing × Crushing stress

$$= l \times \frac{t}{2} \times \sigma_c$$

Therefore, Torque transmitted byy the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{...(ii)}$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} - l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

Or

$$\frac{w}{t} - \frac{\sigma_c}{2\tau}$$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from the above equation, we have w = t. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft. We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2}$$

And torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3$$

From the above

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau}$$

When the key material is same as that of the shaft, then $\tau = \tau_1$. So, l = 1.571 d.

Cottered Joints:

A cotter is a flat wedge shaped piece of rectangular cross-section and its wi dth is tapered (either on one side or both sides) from one end to another for an easy adjustm ent. The taper varies from 1 in 48 to 1 in 24 and it may be increased up to 1 in 8, if a lock ing device is provided. The locking device may be a taper pin or a set screw used on the low er end of the cotter. The cotter is usually made of mild steel or wrought iron. A cotter joint is a temporary fastening and is used to connect rigidly two co-axial rods or bars which are subjjected to axial tensile or compressive forces. It is usually used in connecting a piston rod to the crosshead of a reciprocating steam engine, a piston rod and its extension as a tail or pump rod , strap end of connecting rod etc.

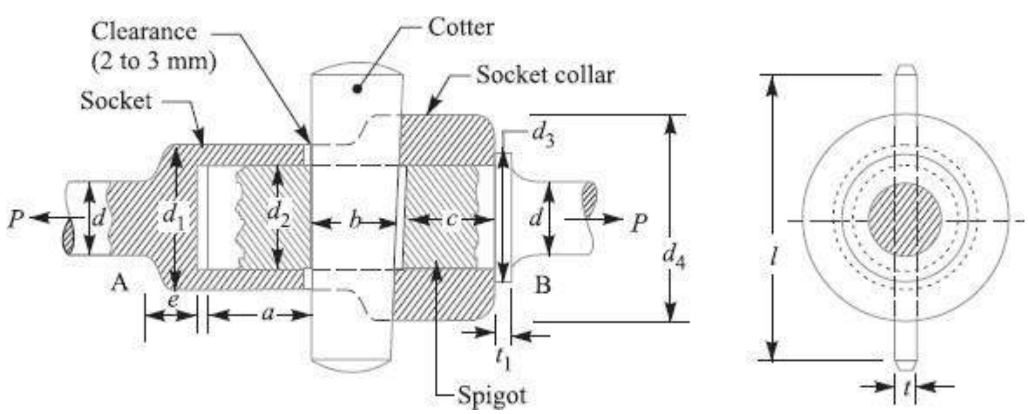
Types of Cotter Joints

Following are the three commonly used cotter joints to connect two rods by a cotter:

1. Socket and spigot cotter joint, **2.** Sleeve and cotter joint, and **3.** Gib and cotter joint.

Socket and Spigot Cotter Joint

In a socket and spigot cotter joinnt, one end of the rods (say *A*) is provided with a socket type of end as shown in Fig., and the other end of the other rod (say *B*) is inserted into a socket. The end of the rod which goes into a socket is also called ***spigot*.** *A* rectangular hole is made in the socket and spigot. *A* cotteer is then driven tightly through a hole in order to make the temporary connection between the two rods. The load is usually acting axially, but it changes its direction and hence the cotter joint must be designed to carry both the tensile and compressive loads. The compressive load is taken up by the collar on the spigot.



Fi g. Socket and spigot cotter joint

Design of Socket and Spigot C otter Joint

The socket and spigot cotter joint is shown in Fig.

Let P = Load carried by the rods,

d = Diameter of the rods,

d_1 = Outside diameter of socket,

d_2 = Diameter of spigot or inside diameter of

socket, d_3 = Outside diameter of spigot collar, t_1= Thickness of spigot collar,

d_4 = Diameter of socket collar,

c = Thickness of socket collar,

b = Mean width of cotter,

t = Thickness of cotter,

l = Length of cotter,

a = Distance from the en d of the slot to the end of rod,

σ_t = Permissible tensile stress for the rods material,

τ = Permissible shear stress for the cotter material, and

σ_c = Permissible crushing stress for the cotter material.

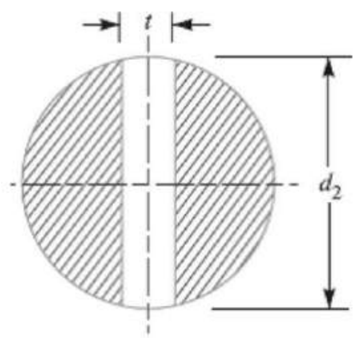
The dimensions for a socket and spigot cotter joint may be obtained by considering the various modes of failure as discussed below:

1. Failure of the rods in tension

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be determined.

2. Failure of spigot in tension across the weakest section (or slot)



$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right] \sigma_t$$

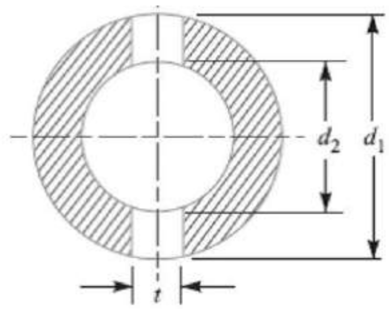
From this equation, the diameeter of spigot or inside diameter of socket (d_2) may be determined. In actual practice, the thickness of cotter is usually taken as d_2 / 4.

3. Failure of the rod or cotter in crushing

$$P = d_2 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

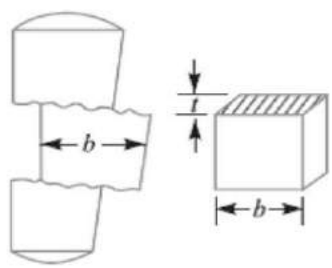
4. Failure of the socket in tension across the slot



$$P = \left\{\frac{\pi}{4}\left[(d_1)^2 - (d_2)^2\right] - (d_1 - d_2)\, t\right\} \sigma_t$$

From this equation, outside diameter of socket (d_1) may be determined.

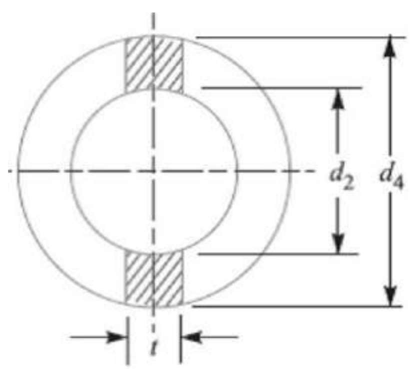
5. Failure of cotter in shear



$$P = 2\, b \times t \times \tau$$

From this equation, width of cotter (b) is determined.

6. Failure of the socket collar in crushing



$$P = (d_4 - d_2)\, t \times \sigma_c$$

From this equation, the diameter of socket collar (d_4) may be obtained.

7. Failure of socket end in shearing

$$P = 2\,(d_4 - d_2)\, c \times \tau$$

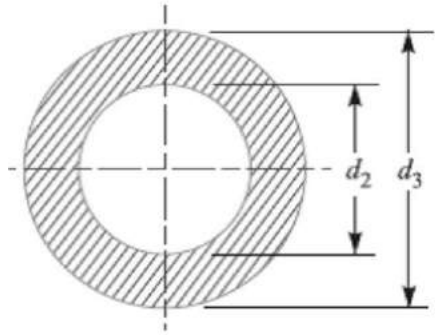
From this equation, the thicknesss of socket collar (c) may be obtained.

8. Failure of rod end in shear

$$P = 2\,a \times d_2 \times \tau$$

From this equation, the distancee from the end of the slot to the end of the rod (a) may be obtained.

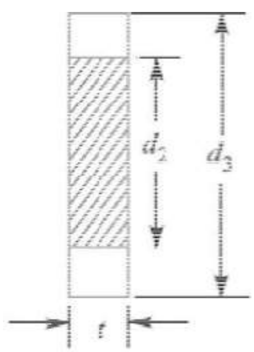
9. Failure of spigot collar in crushing



$$P = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$$

From this equation, the diameter of the spigot collar (d_3) may be obtained.

10. Failure of the spigot collar in shearing

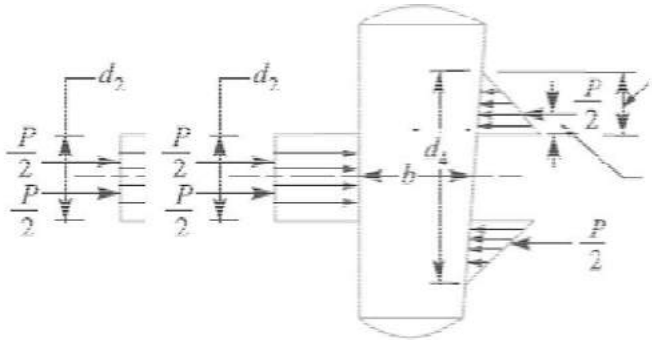


$$P = \pi\, d_2 \times t_1 \times \tau$$

From this equation, the thicknesss of spigot collar (t_1) may be obtained.

11. Failure of cotter in bending

The maximum bending moment occurs at the centre of the cotter and is given by



$$M_{max} = \frac{P}{2}\left(\frac{1}{3} \times \frac{d_4 - d_2}{2} + \frac{d_2}{2}\right) - \frac{P}{2} \times \frac{d_2}{4}$$

$$= \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{2} - \frac{d_2}{4}\right) = \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)$$

We know that section modulus of the cotter,

$$Z = t \times b^2 / 6$$

Bending stress induced in the cotter,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{\frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)}{t \times b^2 / 6} = \frac{P\,(d_4 + 0.5\,d_2)}{2\,t \times b^2}$$

This bending stress induced in the cotter should be less than the allowable bending stress of the cotter.

12. The length of cotter (l) in taken as 4 d.

13. The taper in cotter should no t exceed 1 in 24. In case the greater taper is required, then a locking device must be provided.

14. The draw of cotter is generally taken as 2 to 3 mm.

Notes: 1. when all the parts of the joint are made of steel, the following proportions in terms of diameter of the rod (d) are gennerally adopted:

d_1 = 1.75 d , d_2 = 1.21 d , d_3 = 1.5 d , d_4 = 2.4 d , a = c = 0.75 d , b = 1.3 d, l = 4 d , t = 0.31 d ,t_1 = 0.45 d , e = 1.2 d.

Taper of cotter = 1 in 25, and dra w of cotter = 2 to 3 mm.

2. **If the rod and cotter are made of steel or wrought iron, then $\tau = 0.8\ \sigma_t$ and $\sigma_c = 2\ \sigma_t$** may be taken.

References:

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurm i

3. Design Data hand Book - S MD Jalaludin.

Problem:

Design and draw a cotter joint to support a load varying from 30 kN in compression to 30 kN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied stattically. Tensile stress = compressive stress = 500 MPa ; shear stress = 35 MPa and crushing stress = 90 MPa.

Solution. Given : $P = 30\text{ kN} = 30 \times 10^3\text{ N}$; $\sigma_t = 50\text{ MPa} = 50\text{ N/mm}^2$; $\tau = 35\text{ MPa} = 35\text{ N/mm}^2$; $\sigma_c = 90\text{ MPa} = 90\text{ N/mm}^2$

1. *Diameter of the rods*

Let d = Diameter of the rods.

Considering the failure of the rod in tension. We know that load (P),

$$30 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 50 = 39.3\, d^2$$

$$\therefore \quad d^2 = 30 \times 10^3 / 39.3 = 763 \quad \text{or} \quad d = 27.6 \text{ say } 28\text{ mm} \quad \textbf{Ans.}$$

2. *Diameter of spigot and thickness of cotter*

Let d_2 = Diameter of spigot or inside diameter of socket, and

t = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of spigot in tension across the weakest section. We know that load (P),

$$30 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]50 = 26.8\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 30 \times 10^3 / 26.8 = 1119.4 \quad \text{or} \quad d_2 = 33.4 \text{ say } 34\text{ mm}$$

and thickness of cotter, $t = \dfrac{d_2}{4} = \dfrac{34}{4} = 8.5\text{ mm}$

Let us now check the induced crushing stress. We know that load (P),

$$30 \times 10^3 = d_2 \times t \times \sigma_c = 34 \times 8.5 \times \sigma_c = 289\,\sigma_c$$

$$\therefore \quad \sigma_c = 30 \times 10^3 / 289 = 103.8\text{ N/mm}^2$$

Since this value of σ_c is more than the given value of $\sigma_c = 90\text{ N/mm}^2$, therefore the dimensions $d_2 = 34$ mm and $t = 8.5$ mm are not safe. Now let us find the values of d_2 and t by substituting the value of $\sigma_c = 90\text{ N/mm}^2$ in the above expression, *i.e.*

$$30 \times 10^3 = d_2 \times \frac{d_2}{4} \times 90 = 22.5\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 30 \times 10^3 / 22.5 = 1333 \quad \text{or} \quad d_2 = 36.5 \text{ say } 40\text{ mm} \quad \textbf{Ans.}$$

and $\quad t = d_2 / 4 = 40 / 4 = 10\text{ mm}$ **Ans.**

3. *Outside diameter of socket*

Let d_1 = Outside diameter of socket.

Considering the failure of the socket in tension across the slot. We know that load (P),

$$30 \times 10^3 = \left[\frac{\pi}{4}\left\{(d_1)^2 - (d_2)^2\right\} - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}\left\{(d_1)^2 - (40)^2\right\} - (d_1 - 40)\,10\right]50$$

$$30 \times 10^3 / 50 = 0.7854\,(d_1)^2 - 1256.6 - 10\,d_1 + 400$$

or $(d_1)^2 - 12.7\, d_1 - 1854.6 = 0$

$$\therefore \quad d_1 = \frac{12.7 \pm \sqrt{(12.7)^2 + 4 \times 1854.6}}{2} = \frac{12.7 \pm 87.1}{2}$$

$$= 49.9 \text{ say } 50 \text{ mm } \textbf{Ans.} \qquad \text{...(Taking +ve sign)}$$

4. *Width of cotter*

Let b = Width of cotter.

Considering the failure of the cotter in shear. Since the cotter is in double shear, therefore load (P),

$$30 \times 10^3 = 2\, b \times t \times \tau = 2\, b \times 10 \times 35 = 700\, b$$

$$\therefore \quad b = 30 \times 10^3 / 700 = 43 \text{ mm } \textbf{Ans.}$$

5. *Diameter of socket collar*

Let d_4 = Diameter of socket collar.

Considering the failure of the socket collar and cotter in crushing. We know that load (P),

$$30 \times 10^3 = (d_4 - d_2)\, t \times \sigma_c = (d_4 - 40) 10 \times 90 = (d_4 - 40)\, 900$$

$$\therefore \quad d_4 - 40 = 30 \times 10^3 / 900 = 33.3 \text{ or } d_4 = 33.3 + 40 = 73.3 \text{ say } 75 \text{ mm } \textbf{Ans.}$$

6. *Thickness of socket collar*

Let c = Thickness of socket collar.

Considering the failure of the socket end in shearing. Since the socket end is in double shear, therefore load (P),

$$30 \times 10^3 = 2(d_4 - d_2)\, c \times \tau = 2\,(75 - 40)\, c \times 35 = 2450\, c$$

$$\therefore \quad c = 30 \times 10^3 / 2450 = 12 \text{ mm } \textbf{Ans.}$$

7. *Distance from the end of the slot to the end of the rod*

Let a = Distance from the end of slot to the end of the rod.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$30 \times 10^3 = 2\, a \times d_2 \times \tau = 2a \times 40 \times 35 = 2800\, a$$

$$\therefore \quad a = 30 \times 10^3 / 2800 = 10.7 \text{ say } 11 \text{ mm } \textbf{Ans.}$$

8. *Diameter of spigot collar*

Let d_3 = Diameter of spigot collar.

Considering the failure of spigot collar in crushing. We know that load (P),

$$30 \times 10^3 = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right] \sigma_c = \frac{\pi}{4}\left[(d_3)^2 - (40)^2\right] 90$$

or $$(d_3)^2 - (40)^2 = \frac{30 \times 10^3 \times 4}{90 \times \pi} = 424$$

$$\therefore \quad (d_3)^2 = 424 + (40)^2 = 2024 \text{ or } d_3 = 45 \text{ mm } \textbf{Ans.}$$

9. *Thickness of spigot collar*

Let t_1 = Thickness of spigot collar.

Considering the failure of spigot collar in shearing. We know that load (P),

$$30 \times 10^3 = \pi\, d_2 \times t_1 \times \tau = \pi \times 40 \times t_1 \times 35 = 4400\, t_1$$

$\therefore$ $t_1 = 30 \times 10^3 / 4400 = 6.8$ say 8 mm **Ans.**

10. The length of cotter (l) is taken as 4 d.

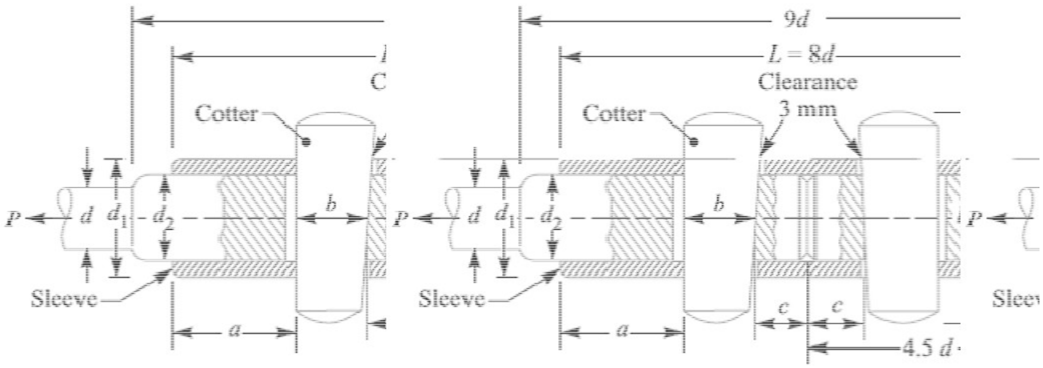
$\therefore$ $l = 4\,d = 4 \times 28 = 112$ mm **Ans.**

11. The dimension e is taken as 1.2 d.

$\therefore$ $e = 1.2 \times 28 = 33.6$ say 34 mm **Ans.**

Sleeve and Cotter Joint

Sometimes, a sleeve and cotter joint as shown in Fig., is used to connect two round rods or bars. In this type of joint, a sleeve or muff is used over the two rods and then two cotters (one on each rod end) are inserted in the holes provided for them in the sleeve and roods. The taper of cotter is usually 1 in 24. It maay be noted that the taper sides of the two cotters should face each other as shown in Fig. The clearance is so adjusted that when the cotters are driven in, the two rods come closer to each other thus making the joint tight.



The various proportions for the sleeve and cotter joint in terms of the diameter of rod (d) are as follows :

Outside diameter of sleeve,

$d_1 = 2.5$ d

Diameter of enlarged end of rod,

d_2 = Inside diameter of sleeve = 1.25 d

Length of sleeve, L = 8 d

Thickness of cotter, t = d2/4 or 0.31 d

Width of cotter, b = 1.25 d

Length of cotter, l = 4 d

Distance of the rod end (a) fro m the beginning to the cotter hole (inside the sleeve end) = Distance of the rod end (c) from its end to the cotter hole = 1.25 d

Design of Sleeve and Cotter Joint

The sleeve and cotter joint is shown in Fig.

Let P = Load carried by the rods,

d = Diameter of the rods,

d_1 = Outside diameter of sleeve,

d_2 = Diameter of the enlarged end of rod,

t = Thickness of cotter,

l = Length of cotter,

b = Width of cotter,

a = Distance of the rod end from the beginning to the cotter hole (inside the sleeve end),

c = Distance of the rod end from its end to the cotter hole,

σ_t , τ and σ_c = Permissible tensile, shear and crushing stresses respectively for the material of the rods and cotter.

The dimensions for a sleeve and cotter joint may be obtained by considering the various modes of failure as discussed below:

1. Failure of the rods in tension

The rods may fail in tension due to the tensile load P. We know that

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be obtained.

2. Failure of the rod in tension across the weakest section (i.e. slot)

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

From this equation, the diameter of enlarged end of the rod (d_2) may be obtained. The thickness of cotter is usually taken as d_2 / 4.

3. Failure of the rod or cotter in crushing

$$P = d_2 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

4. Failure of sleeve in tension across the slot

$$P = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\, t\right]\sigma_t$$

From this equation, the outside diameter of sleeve (d_1) may be obtained.

5. Failure of cotter in shear

$$P = 2b \times t \times \tau$$

From this equation, width of cotter (b) may be determined.

6. Failure of rod end in shear

$$P = 2\,a \times d_2 \times \tau$$

From this equation, distance (a) may be determined.

7. Failure of sleeve end in shear

$$P = 2(d_1 - d_2)\, c \times \tau$$

From this equation, distance (c) may be determined.

Problem:

Design a sleeve and cotter joint to resist a tensile load of 60 kN. All parts of the joint are made of the same material with the following allowable stresses: $\boldsymbol{\sigma}_t$ = 60 MPa ; $\boldsymbol{\tau}$ = 70 MPa ; and $\boldsymbol{\sigma}_c$ = 125 MPa.

Solution. Given : $P = 60$ kN $= 60 \times 10^3$ N ; $\sigma_t = 60$ MPa $= 60$ N/mm^2 ; $\tau = 70$ MPa $= 70$ N/mm^2 ; $\sigma_c = 125$ MPa $= 125$ N/mm^2

1. *Diameter of the rods*

Let d = Diameter of the rods.

Considering the failure of the rods in tension. We know that load (P),

$$60 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 60 = 47.13\, d^2$$

$$\therefore \quad d^2 = 60 \times 10^3 / 47.13 = 1273 \quad \text{or} \quad d = 35.7 \text{ say } 36 \text{ mm} \textbf{ Ans.}$$

2. *Diameter of enlarged end of rod and thickness of cotter*

Let d_2 = Diameter of enlarged end of rod, and

t = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of the rod in tension across the weakest section (*i.e.* slot). We know that load (P),

$$60 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right] 60 = 32.13\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 60 \times 10^3 / 32.13 = 1867 \quad \text{or} \quad d_2 = 43.2 \text{ say } 44 \text{ mm} \textbf{ Ans.}$$

and thickness of cotter,

$$t = \frac{d_2}{4} = \frac{44}{4} = 11 \text{ mm} \textbf{ Ans.}$$

Let us now check the induced crushing stress in the rod or cotter. We know that load (P),

$$60 \times 10^3 = d_2 \times t \times \sigma_c = 44 \times 11 \times \sigma_c = 484\, \sigma_c$$

$$\therefore \quad \sigma_c = 60 \times 10^3 / 484 = 124 \text{ N/mm}^2$$

Since the induced crushing stress is less than the given value of 125 N/mm^2, therefore the dimensions d_2 and t are within safe limits.

3. *Outside diameter of sleeve*

Let d_1 = Outside diameter of sleeve.

Considering the failure of sleeve in tension across the slot. We know that load (P)

$$60 \times 10^3 = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\, t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}[(d_1)^2 - (44)^2] - (d_1 - 44)\, 11\right] 60$$

$$\therefore \quad 60 \times 10^3 / 60 = 0.7854\,(d_1)^2 - 1520.7 - 11\,d_1 + 484$$

or $$(d_1)^2 - 14\,d_1 - 2593 = 0$$

$$\therefore \quad d_1 = \frac{14 \pm \sqrt{(14)^2 + 4 \times 2593}}{2} = \frac{14 \pm 102.8}{2}$$

$$= 58.4 \text{ say } 60 \text{ mm } \textbf{Ans.}$$

4. *Width of cotter*

Let b = Width of cotter.

Considering the failure of cotter in shear. Since the cotter is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,b \times t \times \tau = 2 \times b \times 11 \times 70 = 1540\,b$$

$$\therefore \quad b = 60 \times 10^3 / 1540 = 38.96 \text{ say } 40 \text{ mm } \textbf{Ans.}$$

5. *Distance of the rod from the beginning to the cotter hole (inside the sleeve end)*

Let a = Required distance.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,a \times d_2 \times \tau = 2\,a \times 44 \times 70 = 6160\,a$$

$$\therefore \quad a = 60 \times 10^3 / 6160 = 9.74 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

6. *Distance of the rod end from its end to the cotter hole*

Let c = Required distance.

Considering the failure of the sleeve end in shear. Since the sleeve end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,(d_1 - d_2)\,c \times \tau = 2\,(60 - 44)\,c \times 70 = 2240\,c$$

$$\therefore \quad c = 60 \times 10^3 / 2240 = 26.78 \text{ say } 28 \text{ mm } \textbf{Ans.}$$

GIB AND COTTER JOINT

This joint is generally ussed to connect two rods of square or rectangul ar section. To make the joint; one end of the rod is formed into a U-fork, into which, the end of the other rod fits-in. When a cotter is driven-in, the friction between the cotter and straps of the U-fork, causes the straps open. This is prevented by the use of a gib.

A gib is also a wedge shaped piece of rectangular cross-section with tw o rectangular projections, called lugs. One side of the gib is tapered and the other straight. The tapered side of the gib bears against the tapered side of the cotter such that the outer edges of the cotter and gib as a unit are parallel. This facilitates making of slots with parallel edges, unlike the tapered edges in case of ordinary cotter joint. The gib also provides larger s urface for the cotter to slide on. For making th e joint, the gib is placed in position first, and then the cotter is driven-in.

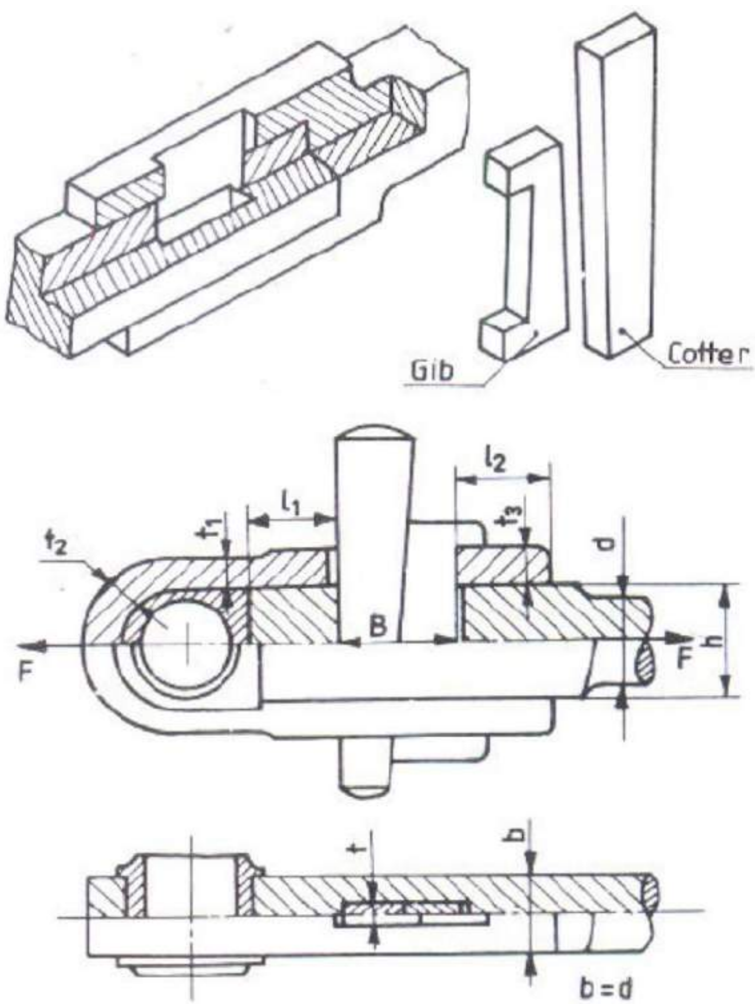


Fig. Gib and cotter Joint

Let *F* be the maximum tensile or compressive force in the connecting rod, and b = width of the strap, which may be taken as equal to the diameter of the rod.

D h = height of the rod end

t_1 = thickness of the strap at the thinnest part

t_2 = thickness of the strap at the curved portion

t_3 =thickness of the strap across the slot

L1 = length of the rod end, beyond the slot

12 = length of the strap, beyond the slot

B = width of the cotter and gib

t = thickness of the cotter

Let the rod, strap, cotter, and gib are m**ade of the same material with σ_c' σ_t'** and **τ** :as the permissible stresses. The following are the possible modes of failure, and the corresponding design equations, which may be considered for the design of the joint:

1. Tension failure of the rod across the section of diameter, *D*

$$F = \frac{\pi d^2}{4} \times \sigma_t$$

2. Tension failure of the rod across the slot(Fig.1)

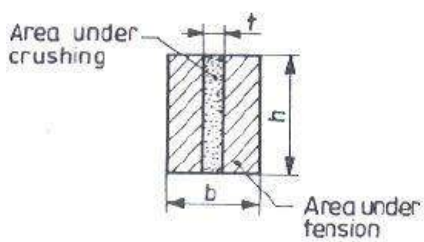


Fig.1

$$F = (bh - ht)\,\sigma_t$$

If the rod and strap are made of the same material, and for equality of strength,

$h=2t_3$ 3. Tension failure of the strap, ac ross the thinnest part (Fig.2)

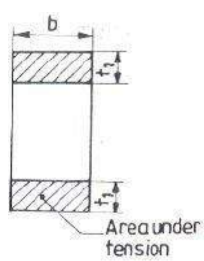


Fig.2

$$F = 2b\,t_1\,\sigma_t$$

4. Tension failure of the strap across the slot (Fig.3)

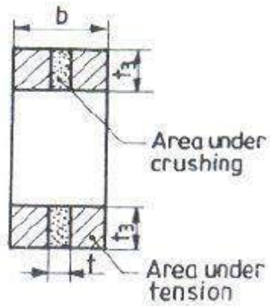


Fig.3

$$F = 2\,b\,t_3 - 2\,t\,t_3 = 2t_3(b - t)\,\sigma_t$$

The thickness, t2 may be taken as (1.15 to 1.5) t], and

Thickness of the cotter, t = b/4.

5. Crushing between the rod and cotter (Fig.1)

$$F = h\,t\,\boldsymbol{\sigma}_c \text{ ; and } h = 2t_3$$

6. Crushing between the strap and gib(Fig.3)

$$F = 2\,t\,t_3\,\boldsymbol{\sigma}_c$$

7. Shear failure of the rod end. It is under double shear (Fig.4).

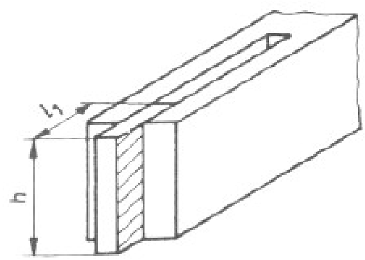


Fig.4

$$F = 2l_1\mathbf{h}\boldsymbol{\tau}$$

8. Shear failure of the strap end. It is under double shear (Fig.5).

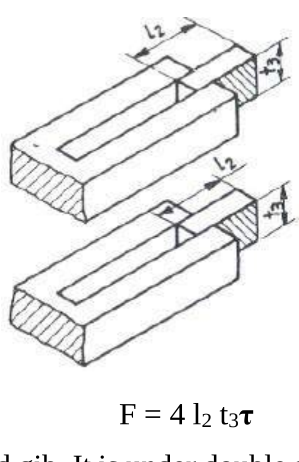


Fig.5

$$F = 4\,l_2\,t_3\boldsymbol{\tau}$$

9. Shear failure of the cotter and gib. It is under double shear.

$$\mathbf{F=2Bt}\boldsymbol{\tau}$$

The following proportions for the widths of the cotter and gib may be followed:

Width of the cotter =0.45 B

Width of the gib = 0.55 B

The above equations may be solved, keeping in mind about the various relations and proportions suggested.

PROBLEM:

Design a cotter joint to connect piston rod to the crosshead of a double acting steam engine. The diameter of the cylinder is 300 mm and the steam pressure is 1 N/mm^2. The allowable stresses for the material of cotter and piston rod are as follows: σ_t = 50 MPa ; τ = 40 MPa ; and σ_c = 84 MPa

Solution. Given : $D = 300$ mm ; $p = 1$ N/mm^2 ; $\sigma_t = 50$ MPa $= 50$ N/mm^2 ; $\tau = 40$ MPa $= 40$ N/mm^2 ; $\sigma_c = 84$ MPa $= 84$ N/mm^2

We know that maximum load on the piston rod,

$$P = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4}(300)^2\ 1 = 70\ 695\ \text{N}$$

The various dimensions for the cotter joint are obtained by considering the different modes of failure as discussed below :

1. *Diameter of piston rod at cotter*

Let d_2 = Diameter of piston rod at cotter, and

t = Thickness of cotter. It may be taken as $0.3\ d_2$.

Considering the failure of piston rod in tension at cotter. We know that load (P),

$$70\ 695 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - 0.3\ (d_2)^2\right]50 = 24.27\ (d_2)^2$$

$\therefore$ $(d_2)^2 = 70\ 695 / 24.27 = 2913$ or $d_2 = 53.97$ say 55 mm **Ans.**

and $t = 0.3\ d_2 = 0.3 \times 55 = 16.5$ mm **Ans.**

2. *Width of cotter*

Let b = Width of cotter.

Considering the failure of cotter in shear. Since the cotter is in double shear, therefore load (P),

$$70\ 695 = 2\ b \times t \times \tau = 2\ b \times 16.5 \times 40 = 1320\ b$$

$\therefore$ $b = 70\ 695 / 1320 = 53.5$ say 54 mm **Ans.**

3. *Diameter of socket*

Let d_3 = Diameter of socket.

Considering the failure of socket in tension at cotter. We know that load (P),

$$70\ 695 = \left\{\frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right] - (d_3 - d_2)\ t\right\}\sigma_1$$

$$= \left\{\frac{\pi}{4}\left[(d_3)^2 - (55)^2\right] - (d_3 - 55)\ 16.5\right\}50$$

$$= 39.27\ (d_3)^2 - 118\ 792 - 825\ d_3 + 45\ 375$$

or $(d_3)^2 - 21\, d_3 - 3670 = 0$

$$\therefore \quad d_3 = \frac{21 + \sqrt{(21)^2 + 4 \times 3670}}{2} = \frac{21 \pm 123}{2} = 72 \text{ mm} \quad \text{...(Taking + ve sign)}$$

Let us now check the induced crushing stress in the socket. We know that load (P),

$$70\ 695 = (d_3 - d_2)\, t \times \sigma_c = (72 - 55)\, 16.5 \times \sigma_c = 280.5\, \sigma_c$$

$$\therefore \quad \sigma_c = 70\,695 / 280.5 = 252 \text{ N/mm}^2$$

Since the induced crushing is greater than the permissible value of 84 N/mm^2, therefore let us find the value of d_3 by substituting σ_c = 84 N/mm^2 in the above expression, *i.e.*

$$70\ 695 = (d_3 - 55)\, 16.5 \times 84 = (d_3 - 55)\, 1386$$

$$\therefore \quad d_3 - 55 = 70\,695 / 1386 = 51$$

or $$d_3 = 55 + 51 = 106 \text{ mm Ans.}$$

We know the tapered length of the piston rod,

$$L = 2.2\, d_2 = 2.2 \times 55 = 121 \text{ mm Ans.}$$

Assuming the taper of the piston rod as 1 in 20, therefore the diameter of the parallel part of the piston rod,

$$d = d_2 + \frac{L}{2} \times \frac{1}{20} = 55 + \frac{121}{2} \times \frac{1}{20} = 58 \text{ mm Ans.}$$

and diameter of the piston rod at the tapered end,

$$d_1 = d_2 - \frac{L}{2} \times \frac{1}{20} = 55 - \frac{121}{2} \times \frac{1}{20} = 52 \text{ mm Ans.}$$

DESIGN OF KNUCKLE JOINNT

The following figure shows a knuckle joint with the size parameters an d proportions indicated. In general, the rods coonnected by this joint are subjected to tensile loads, although if the rods are guided, they may support compressive loads as well. Let F. = tensile load to be resisted by the joint

d = diameter of the rods

d_1 = diameter of the knuckle pin

D = outside diameter of the eye

A =thickness of the fork

B =thickness of the eye

Obviously, if the rods are made of the same material, the parameters, *A* and *B* are related as,

B=2A

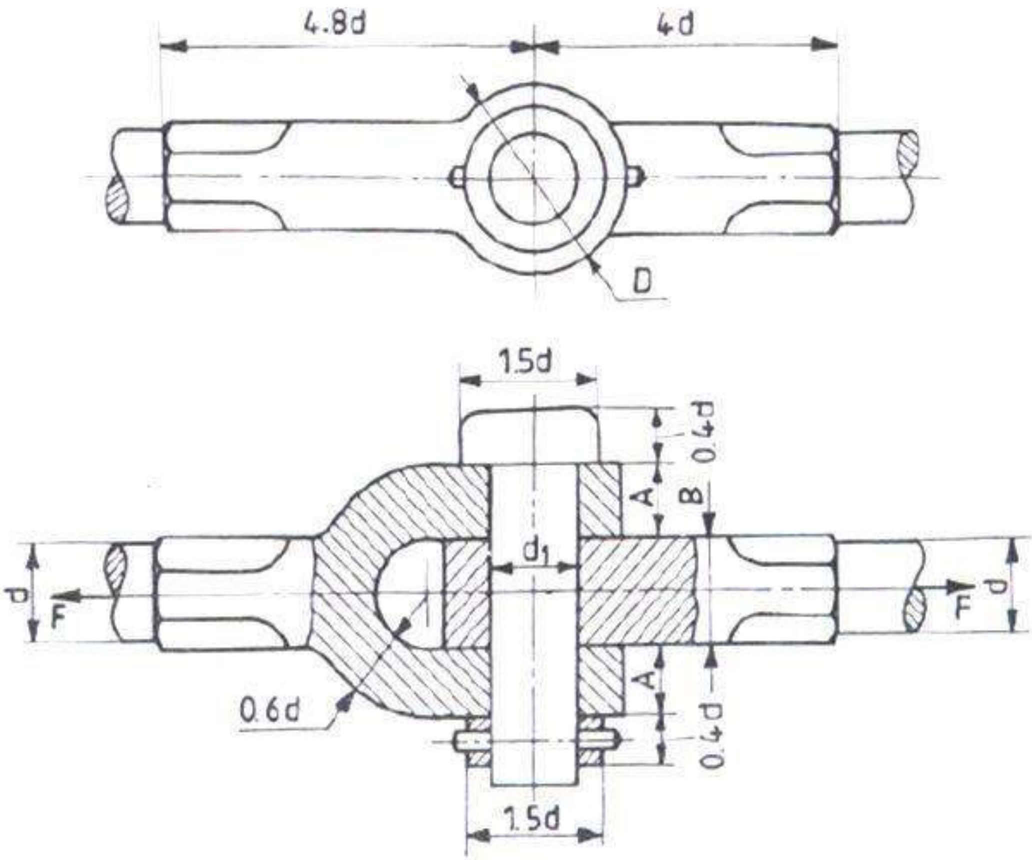


Fig. Knuckle Joint

Let the rods and pin are made of the same material, with **σ_t, σ_c and τ** as the permissible stresses. The following are the possible modes of failure, and the corresponding design equations, which may be conside red for the design of the joint:

1. Tension failure of the rod, across the section of diameter, *D*

F $\frac{D^2}{4}$

2. Tension failure of the eye (fig.1)

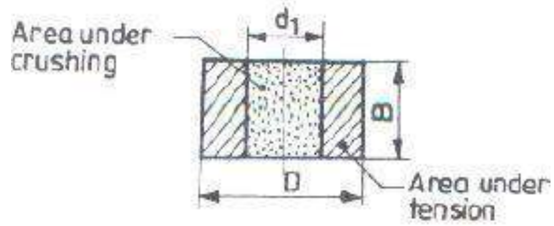


Fig.1

$$F = (D-d_1)\, \mathbf{B}\, \boldsymbol{\sigma}_t$$

3. Tension failure of the fork (fig .2)

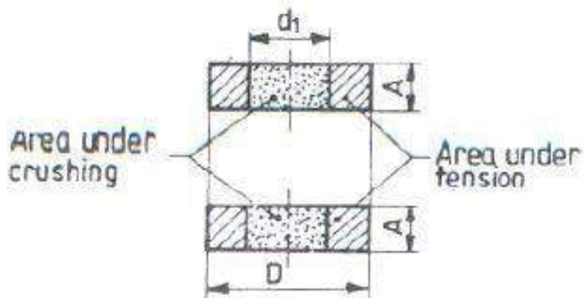


Fig.2

$$F = 2\,(D - d_1)\, \mathbf{A}\, \boldsymbol{\sigma}_t$$

4. Shear failure of the eye (Fig.3)

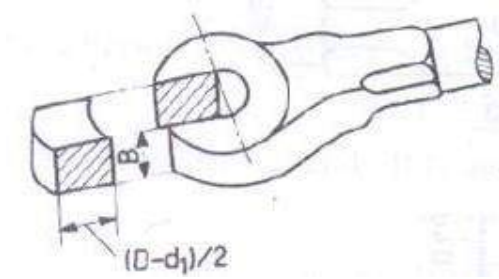


Fig.3

$$F = (D-d_1)\, \mathbf{B}\, \boldsymbol{\tau}$$

5. Shear failure of the fork (Fig.4)

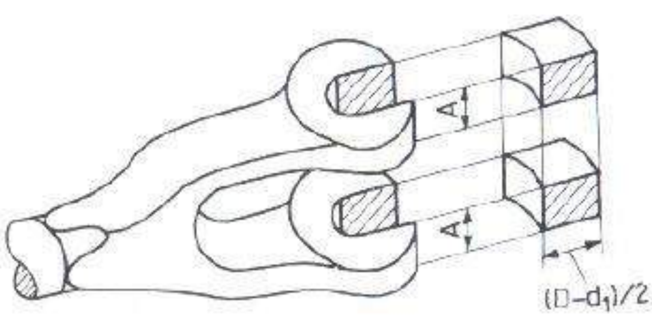


Fig.4

$$F = 2\,(D-d_1)\, \mathbf{A}\, \boldsymbol{\tau}$$

6. Shear failure of the pin. It is under double shear.

F 2X D^2X

4

7. Crushing between the pin and eye (fig.1)

$$F = d_1 \mathbf{B} \boldsymbol{\sigma}_c$$

8. Crushing between the pin and fork (fig.2)

$$F = 2 d_1 \mathbf{A} \boldsymbol{\sigma}_c$$

For size parameters, not covere d by the above design equations; proportions as indicated in the figure may be followed.

Problem:

Design a knuckle joint to transmmit 150 kN. The design stresses may be taken as 75 MPa in tension, 60 MPa in shear and 15 0 MPa in compression.

Solution. Given : $P = 150$ kN $= 150 \times 10^3$ N ; $\sigma_t = 75$ MPa $= 75$ N/mm^2 ; $\tau = 60$ MPa $= 60$ N/mm^2 ; $\sigma_c = 150$ MPa $= 150$ N/mm^2

1. *Failure of the solid rod in tension*

Let d = Diameter of the rod.

We know that the load transmitted (P),

$$150 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 75 = 59\, d^2$$

$\therefore$ $d^2 = 150 \times 10^3 / 59 = 2540$ or $d = 50.4$ say 52 mm **Ans.**

Now the various dimensions are fixed as follows :

Diameter of knuckle pin,

$$d_1 = d = 52 \text{ mm}$$

Outer diameter of eye, $d_2 = 2\,d = 2 \times 52 = 104$ mm

Diameter of knuckle pin head and collar,

$$d_3 = 1.5\,d = 1.5 \times 52 = 78 \text{ mm}$$

Thickness of single eye or rod end,

$$t = 1.25\,d = 1.25 \times 52 = 65 \text{ mm}$$

Thickness of fork, $t_1 = 0.75\,d = 0.75 \times 52 = 39$ say 40 mm

Thickness of pin head, $t_2 = 0.5\,d = 0.5 \times 52 = 26$ mm

2. *Failure of the knuckle pin in shear*

Since the knuckle pin is in double shear, therefore load (P),

$$150 \times 10^3 = 2 \times \frac{\pi}{4} \times (d_1)^2\, \tau = 2 \times \frac{\pi}{4} \times (52)^2\, \tau = 4248\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 4248 = 35.3 \text{ N/mm}^2 = 35.3 \text{ MPa}$$

3. *Failure of the single eye or rod end in tension*

The single eye or rod end may fail in tension due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, t \times \sigma_t = (104 - 52)\, 65 \times \sigma_t = 3380\, \sigma_t$$

$$\therefore \quad \sigma_t = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

4. *Failure of the single eye or rod end in shearing*

The single eye or rod end may fail in shearing due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, t \times \tau = (104 - 52)\, 65 \times \tau = 3380\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

5. *Failure of the single eye or rod end in crushing*

The single eye or rod end may fail in crushing due to the load. We know that load (P),

$$150 \times 10^3 = d_1 \times t \times \sigma_c = 52 \times 65 \times \sigma_c = 3380\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

6. *Failure of the forked end in tension*

The forked end may fail in tension due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, 2\, t_1 \times \sigma_t = (104 - 52)\, 2 \times 40 \times \sigma_t = 4160\, \sigma_t$$

$$\therefore \quad \sigma_t = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

7. *Failure of the forked end in shear*

The forked end may fail in shearing due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, 2\, t_1 \times \tau = (104 - 52)\, 2 \times 40 \times \tau = 4160\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

8. *Failure of the forked end in crushing*

The forked end may fail in crushing due to the load. We know that load (P),

$$150 \times 10^3 = d_1 \times 2\, t_1 \times \sigma_c = 52 \times 2 \times 40 \times \sigma_c = 4160\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 4180 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

From above, we see that the induced stresses are less than the given design stresses, therefore the joint is safe.