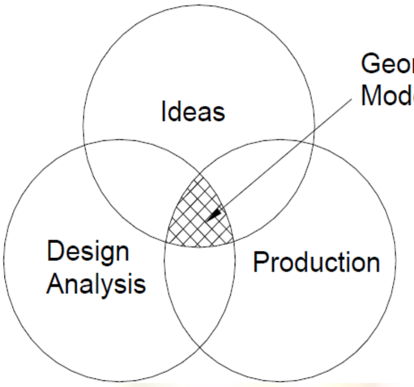
UNIT-II
GEOMETRIC MODELING

INTRODUCTION



Geometric modeling is a branch of applied mathematical and computational geometry that studies methods and algorithms for mathematical description of shapes

The shapes studied in Geometric modeling are mostly 2-Dimensional or 3- Dimensional although many of its tools and principles can be applied to sets of any finite dimension. Most Geometric modeling is done with computer and computer based applications. 2-Dimensions models are important in computer typography (The art and technique of arranging type in order to make language visible) and technical drawings. 3- Dimensional models are central to CAD and CAD many applied technical fields. Such as civil, mechanical, architecture, geology, medical image processing.

NEED OF GEOMETRIC MODELING

For manufacturing automobile engine it require around 10,000 drawings models in various departments with different styles. Once models are prepared using any CAD software, these models can be used in almost all departments of the organization which includes-

1. Designing the parts and assemblies and then performing tolerance analysis of theassemblies
2. Preparation of production drawings of individual parts, subassemblies, assemblies,tooling and jig-fixtures
3. Making structural, thermal and kinetic analysis
4. Considering volume of parts and after extracting material properties from material library,cost estimation can be easily done

5. Process plans are also prepared for all the components
6. Using CAM software's, we can model as input for CNC programming
7. Programming the movement of the components from one station to another is possibleusing various material devices such as robots, conveyors, automated guided vehicles
8. Using coordinate measuring machine (CMM) dimensional and geometric accuracies arechecked by extracting various dimensions and parametric from the models only
9. Other supporting activities such as material requirement and procurement, preparation ofbill of materials, manufacturing resource requirement, planning and scheduling, analysis

With competitions products etc are also possible once geometric models are available thus for all these activities, geometric models becomes the central that will be manipulate at all these storages. While modeling, geometric models are prepared in such a way that the other modules such as CAM, CAE, Mechanism systems are able to use this information in the most optimal and efficient way.

REQUIREMENTS OF GEOMETRIC MODELING

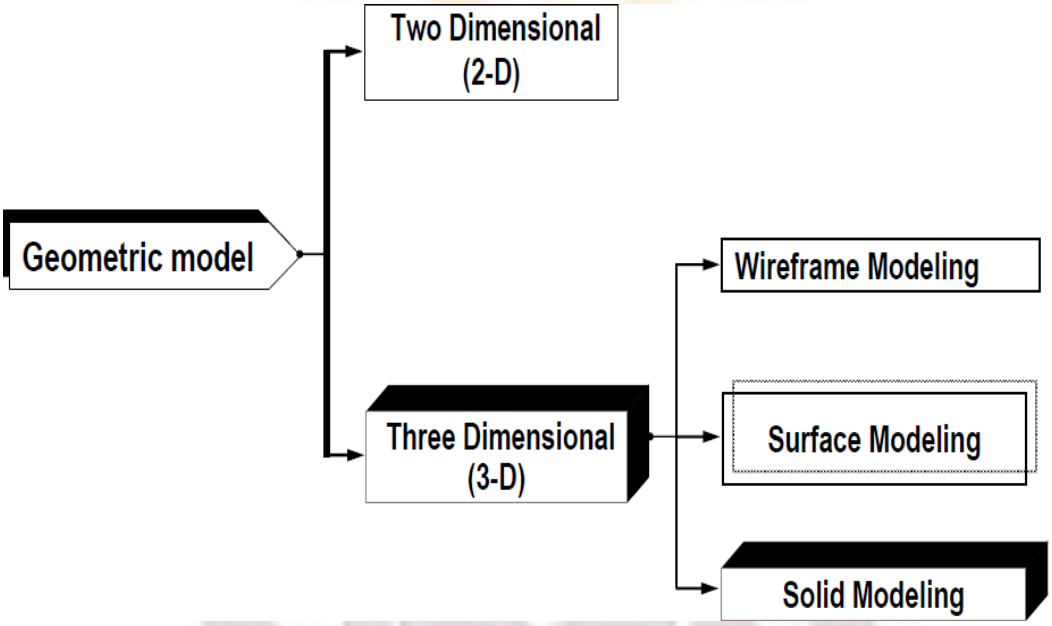
The requirement of geometric modeling is manifold. The conceptual design is the basis of the generation of geometric model. The choice of the geometric model depends on the mechanical functions to be performed by it. A valid geometric model is created by CAD system and its model database is stored. The database of geometric modeling is used for engineering analysis and for design optimization. Design testing and evaluation may necessitate changing the geometric model before finalizing it. When the final design isachieved, it is documented and used for subsequent manufacturing applications, quality and cost analysis.
Good geometric model is designed based in the following important guidelines-

1. Modeling method must be easy to use
2. Completeness of part representation
3. Representation should be able to represent a useful set of geometric objects
4. Model should not create any ambiguity (The ability to express more than one interpretation) to users. A given representation should correspond to one and only one solid
5. To represent the solid normally there should only unique way. With this one can compare two identical solid representations
6. A geometric model must be very accurate. There should not be any approximation
7. Geometric modeling should not create any invalid or impossible models

8. The various transformations such as move, rotation, scale etc and manipulations such asunion, intersect, subtract etc should able to be performed on geometric model
9. A good representation should be compact enough for saving space and allow for efficientalgorithms to determine desired physical characteristics
10. Geometric model should have shading and rendering (Generation an image for a modelby means of a computer program) capability to give realistic effect to the model

GEOMETRIC MODELS

The geometric models are broadly classified on the basis of geometric construction into the following categories



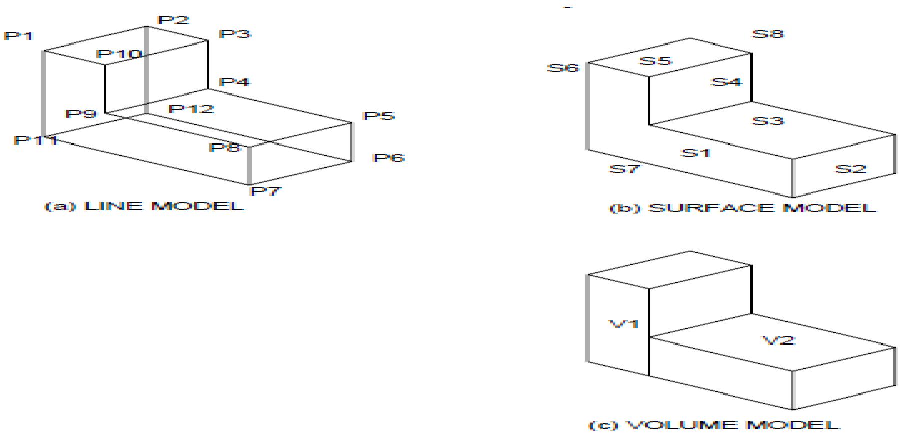
1. Two Dimensional (2D)
2. Three Dimensional (3D)

Two Dimensional (2D)

The 2D modeling includes the construction of geometrical faces, plane drawings, 2D views (Top, Front, Right, and Light views) of objects. Presently, the application of 2D model is limited to drafting, sheet metal manufacturing, Spot welding, Laser cutting etc.

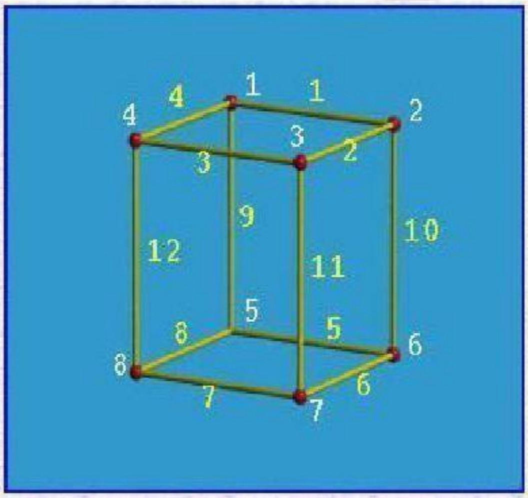
Three dimensional (3D)

The 3D modeling is widely used for engineering applications. It provides all the information required for animation, design analysis and manufacturing. The 3D objects are shown in below figure.



The 3D models are further sub-divided into **three** groups:

1. Wireframe modeling



Edge Table		
Edge #	*Start Vertex*	*End Vertex*
1	1	2
2	2	3
3	3	4
4	4	1
5	5	6
6	6	7
7	7	8
8	8	5
9	1	5
10	2	6
11	3	7
12	4	8

Vertex Table			
Vertex #	*x*	*y*	*z*
1	1	1	1
2	1	-1	1
3	-1	-1	1
4	-1	1	1
5	1	1	-1
6	1	-1	-1
7	-1	-1	-1
8	-1	1	-1

Wireframe modeling is the oldest and simplest methods of geometric modeling which can be used to store model mathematically in the computer memory. It contains information about the locations of all the points (vertices) and edges in space coordinates. Various wireframe entities are points, lines, planer arcs, circle, curves etc. Each vertex is defined by x, y, z coordinates. Edges are defined by a pair of vertices and faces are defined as three or more edges. Thus wireframe is a collection of edges, there is no skin defining the area between the edges. This is the lowest level of modeling and has serious limitations. But it some applications such as tool path simulation it is very convenient to use wireframe models.

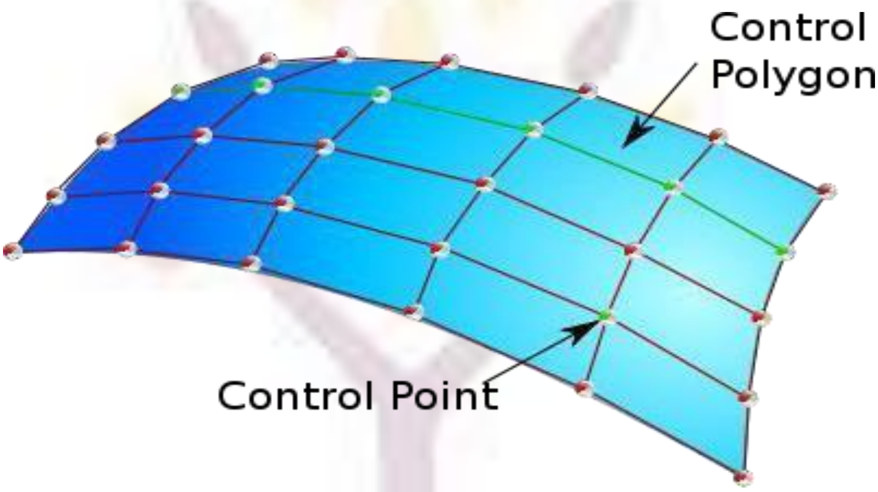
Advantages of wireframe models:

1. It is simple methods and requires less memory space
2. It forms the basis for surface and solid modeling
3. Manipulations in the model can be done easily and quickly

Disadvantages:

1. One of the serious limitation in the ambiguity of orientation and viewing plane
2. Cannot model complex curve surfaces
3. Does not represent an actual solid (no surface and volume)
4. Physical properties such as mass, surface area, volume, centre of gravity etc are not possible to calculate
5. Wireframe models has no knowledge of surface faces, therefore it will not detect interface between two matting components and this is serious drawback especially in component assembly, kinematic analysis, NC tool and robot arm simulation

2. Surface modeling



Surface modeling is the next stage of wireframe modeling. In wireframe modeling models are unable to represent complex surfaces of objects like car, ship, aeroplane, wings, castings etc. only a surface profile of these objects. A surface model represents the skin of an object. These skins have no thickness or material type. Surface models define the surface properties, as well as the edges of objects. These are often capable of clearly representing the solid from the manufacturing. However, no information regarding the interior of the solid model would be available which could be relevant for generating the NC cutter data. Further the calculation of properties such as mass and inertia etc would be difficult. Surface modeling facilities would be available as part of the modeling technique and would be used when such surface is present in the product for design. For example this method is used mode for specific non-analytical surfaces, called sculptures surfaces such as those used for modeling the car bodies and ship-hulls. There are a number of mathematical techniques available for handling these surfaces such as Bezier and B-splines.

Advantage:

1. Eliminates much ambiguity and non-uniqueness present in wireframe models by hidinglines not seen
2. Renders the model for better visualization and presentation, objects appear more realistic
3. Provides the surface geometry for CAM, NC machine
4. Provides the geometry needed by the manufacturing engineer for mould and die design
5. This can be used to design and analysis complex free-formed surfaces of ship hulls,aeroplane fuselages and bodies
6. Surface properties such as roughness, color and reflectivity can be assigned anddemonstrated

Disadvantages:

1. Provides no information about the inside of an object
2. Curved surfaces need a fine mesh to be accurate
3. Provides wrong results if mesh is too coarse
4. Complicated computation, depending on the number of surfaces

3. Solid modeling

Solid modeling is the most powerful of 3D modeling technique. This includes vertices (nodes), edges, surfaces, weight and volume. This model consisting of the complete description of the solid in a certain form is the most ideal representation, as all the information requires at every stage of product cycle can be obtained with technique. Defining an object with a solid model is the easiest of the available three modeling techniques. The model is a complete and unambiguous representation of a precisely enclosed and filled volume. Solid model contain bath geometric and available to represent the solid, but geometry two techniques are very famous, these are constructive solid geometry (CSG) and boundary representation (B-rep)

Advantages:

1. Mass properties such as area, volume, weight, centre of gravity and moment of inertia canbe determined quickly
2. It allows the design engineer to develop and evaluate alternative concepts for parts andassemblies while the design is still a theoretical model.
3. Solid models are non-ambiguous
4. Easily exported to different FEM programs for analysis
5. It can be used in newly manufacturing techniques; CIM, CAM, design for manufacturing (DFM)

6. 2D standard drawings, assembly drawings and exploded drawings are generated from themodel

Disadvantage:

1. More intensive computation than wireframe and surface modeling
2. Requires more powerful computers (faster with the more memory)

DIFFERENCE AMONG WIREFRAME, SURFACE AND SOLID MODELING

Parameter	Wireframe model	Surface model	Solid model
1. Computer memory	Less	Moderate	Large
2. Entities used	Points, line, circle, arc, ellipse, synthetic curves, such as Bezier, hermite, B-spline etc.	Plane, revolve, ruled, tabulated, free form surfaces	Solid primitives as cone, cube, wedge, cylinder, sphere etc
3. Input data required	More	Moderate	Less
4. Automatic orthographic, perspective, isometricview generation	Impossible	Impossible	Easily possible
5. NC code generation	Not possible	Automatic possible	Automatic possible
6. Interference between matting parts	Not possible to detect	Can detect	Can detect
7. Rendering and shadow effect	Not possible	possible	possible
8. Cross sectioning	Not possible donemanually	Not possible	Possible done automatically
9. Elimination of hidden lines	Done manually	May be possible	Possible
10. Calculation of physical properties such as volume, surface area, center of gravity, M.I etc	Not possible	Possible to calculate some properties	Possible to calculate all properties required for analysis
11. Design parameters optimization	Not possible	Not possible	Possible
12. Generation of assemblyand detail drawings from model and vice versa	Not possible	Not possible	Possible
13. CAD/CAM/CAE modules	Drafting	Drafting, design	Drafting, design, manufacturing, analysis, assembly, mechanism, optimization

MODELING FACILITIES

The total modeling facilities that one would look for in any system can be broadlycategorized as follows:

- The geometric modeling features
- The editing or manipulation features
- The display control facilities
- The drafting facility
- The programming facility
- The analysis features
- The connecting features

Geometric Modeling Features

The various geometric modeling and construction facilities that one should expect to have in any good system are as follows:

1. Various features to aid geometric construction methods, such as Cartesian and polar coordinates, absolute and incremental dimensions, various types of units, grip, snap, object snap, layer etc
2. All 2D analytical features, such as points, lines, arcs, circles, coins, splines, fillets, chamfers etc. In each of these features, various constructional features including interactive and dynamic dragging facilities
3. Majority of the 3D wireframe modeling facilities includes 3D lines, 3D faces, ruled surfaces, linear sweep from 2D topology with any sweep direction, rotational sweep and tapered sweep. General sweep with twist. Rotational about an axis or radial offset for generating helical or spiral shapes
4. Solid modeling with various basic primitives such as block, cylinder, sphere, cone, prism, torus, pyramid, quadrilateral, along with the ability to apply the Boolean operation on any solid that can be constructed using the other techniques available in the modeler
5. Skinning around regular and arbitrary surface. Profiles (cross-sections), both analytical and arbitrary places across any 3D curve
6. Sculptured surfaces of the various types like Bezier, Coons and other free form surfaces

7. Comprehensive range of transformation facilities for interactively assembling the various solid models generated by the modeler with features such as surfaces filling and trimming

Editing or Manipulation Features

These set of facilities refer to the way the geometric data, once created, would be used to advantage for further modeling. Using these facilities, it would be possible to use the geometry created earlier to complete the modeling, thus improving the productivity of the designer. The facilities designed in this category are:

1. Transformation such as move, copy, rotate, scale, elongate or compress, mirror or to any arbitrary coordinate frame
2. The editing features used to alter the already drawn geometric entities, such as stretching, trimming or trimming to any intersection, delete or erase, undo or redo
3. Symbols in drawing refer to often-repeated together set in number of drawings, which may consist of a number of geometric entities that are grouped together and stored as a symbol. This symbol can be recalled at any scale, at any angle or exploded if necessary to treat all of them as separate entities. Symbols can also be of parametric type so that a large variation in symbols can be done without much effort
4. Some of editing facilities are : resizing, relocating and duplicating, filleting and chamfering, windowing, clipping and zooming, exploding, mirroring, lengthening and shortening, renaming named objects, editing solid

Display Control Facilities

In this range of features are all the facilities needed for interacting with the modeling system so as to obtain the necessary feedback at the right time during the modeling stage. Thefacilities required are:

Window - to identify a set of entities for any possible display or editing function

Zoom - to change the scale of display of the image selected in the screen

Pan - to move the image on screen without changing the scale at which the drawing isdisplayed on the screen

Hidden - to remove hidden lines or hidden surfaces for viewing the geometry inproper form

Shading - to show the 3D view of the image on screen complete with the light sourcelocation and the resulting light and shade as it appears on the image

Animation - is the display of a number of images in sequence to imitate the actualmotion of the part

*Clippin*g - helps in discarding the part of the geometry outside the viewing window, such that all the transformations that are to be carries out for zooming and panning ofthe image on the screen are applied only on the necessary geometry. This improves the response of the system.

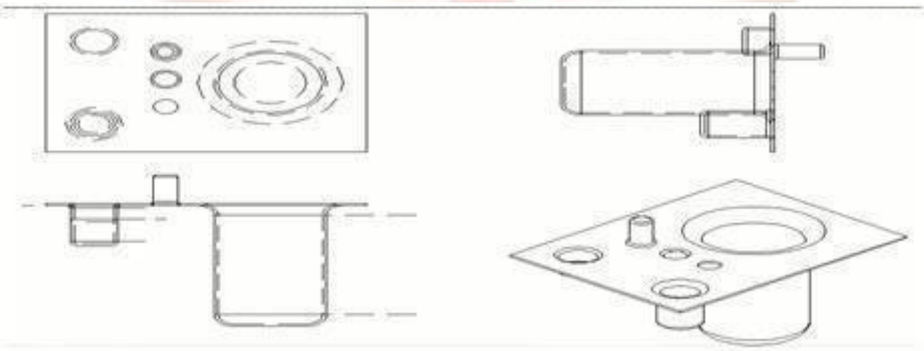
Some other facilities that are required are: *isometric views, sectioning, orthographicviews, perspective views*

Drafting Features

These facilities refer to the way the model developed can be utilized for purpose of transmitting the information in hard copy form for other applications, such as par prints onto the shop floor or maintenance manuals for the equipment. A really large range of facilities are required in this particular category and it is sometimes treated as a separate module in the modeling system

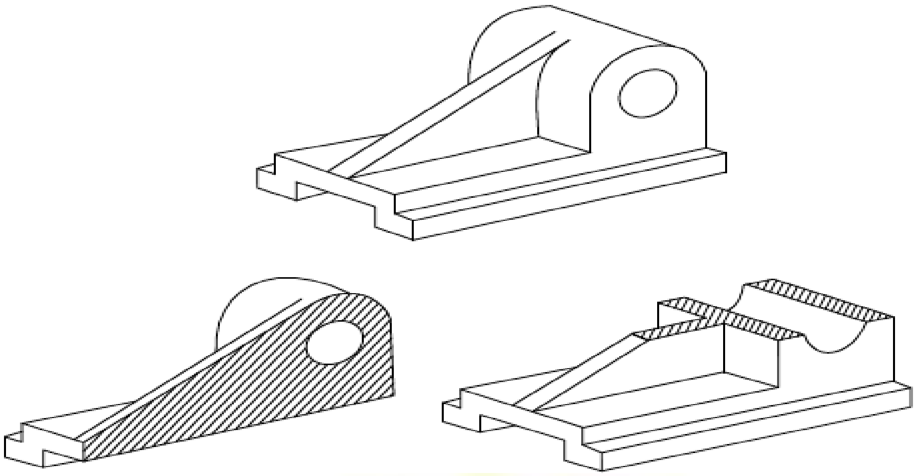
The ability to get various types of lines drawn and provide ample notes in the form of text addition at various locations in the drawing. The text handling capability in terms of font changing and different methods of text presentation should be available.

A large number of types of views should be obtained from the solid model of the geometry stored in the database. The types of views required may be as for display functions, such as perspective views, orthographic views (Shown in below figure), isolated views and axonometric views.



Orthographic views from a Geometric model

It is necessary that the views being shown should be sectioned to get a better appreciation of the model. For this purpose, the section planes may be simple or complex orientations. After sectioning the system should have the automatic ability to show the sectioning details (Shown in below figure) in the form of typical crosshatching depending upon the standard practice.



Programming Facility

Programming ability (MACRO programming) within a CAD system is going to be a very useful feature. It is well known that not all kinds of facilities would be available in any general-purpose CAD system. Therefore, it is necessary that the CAD system would have to be customized for a given range of application process specific to the company. For this purpose, if a programming facility exists in a CAD system, it is possible to program specifically for an application, making use of all the features available in the system for either modeling or for any specific application based on the information generated during the modeling. Some such examples are the GRIP in unigraphics and GLUE in CAM-X. The availability of such a program helps the user to input the least amount of information for any required design, if the application programs are written well using the programming language.

Analysis And Optimization Features

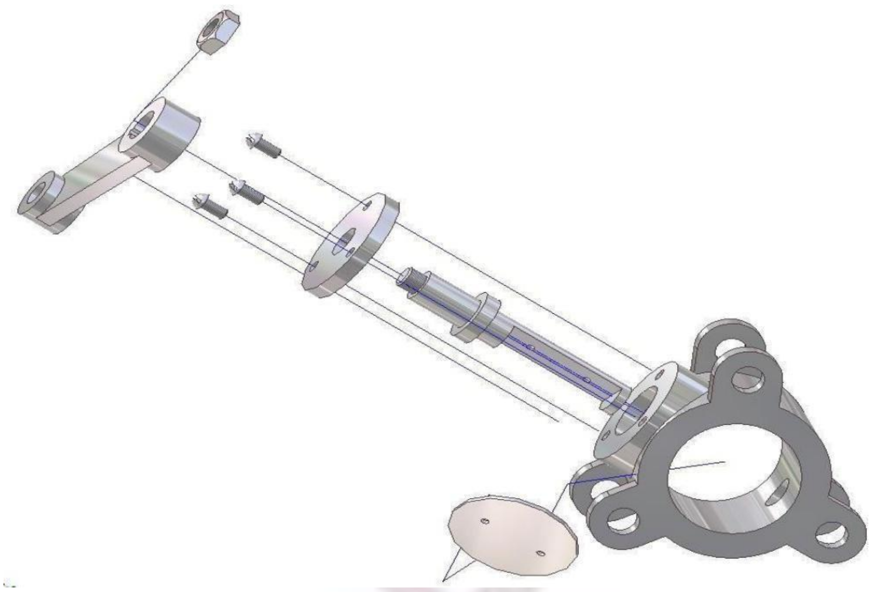
In this range, the kind of analysis facilities that are require to e carried on the product models being generated should be considered. The simplest kind to the most sophisticated features may be available under this category. The simplest facilities may be calculated perimeter, area, volume, mass, centre of gravity, moment of inertia, radius of gyration etc.

Besides these simple features of analysis, a general-purpose analysis that is normally carried is the Finite Element Analysis (FEA). The geometric model created as above could be conveniently passed onto the FEA through an intermediate processor called a Finite Element Methods (FEM), which converts the geometric data into the finite element mesh and calculates all the data required for the analysis and then transmits it to the FEA program. Examples are the SUPERTAB for GEOMOD and the GFEM for the unigraphics.

Another important feature essential in the modeling system used by the mechanical engineering industries is the assembly facility with the associated interference checking. By this, products individually modeled can be assembly joints are analyzed. This would be further used along with animation facility, if present, to see the performance of the assembly in service. Along with the assembly facility, the other facility needed is the ability to explode an assembly (Show in above figure) for the creation of technical illustrations for the user and maintenance manual preparation.

Connecting Features

Modeling is only the start of the complete process of a product evolution and as such the data generated



is used directly by the other systems. It is therefore, necessary that the internal data format in which the data is stores by the modeling system should be well documented and should also have very good connectivity (data interfacing) with other allied modules. Identically, an integrated data base structure would be useful where in all the various modules share the common database. This would only be possible if all the modules are developed at a single developer as in the case of ProEngineer or Unigrapchis for CAD/CAM integration.

GEOMETRIC CONSTRUCTION METHODS

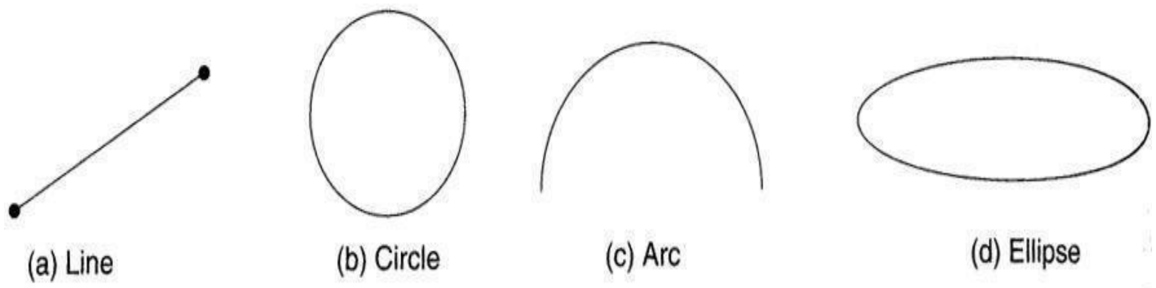
The three-dimensional construction methods are:

- Wireframe modeling
- Surface modeling
- Solid modeling
- Extrusion

- Sweeping
- Feature modeling
- Lofting
- Tweaking

Wireframe Modeling

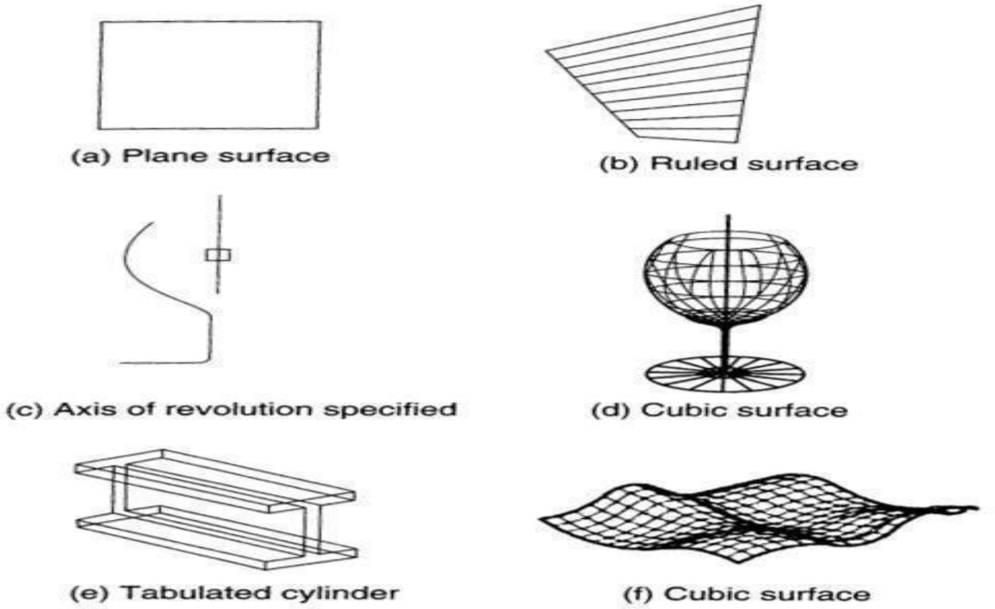
Wireframe modeling uses geometric primitives for the construction of models. The geometric primitives are points, lines, arcs and circles, conies, cubic curve, Bezier curve and B-spline curve.



Wireframe primitives.

Surface Modeling

Surface models are generated by using surface primitives such as plane surface, ruled surface, surface of revolution, tabulated cylinder, fillet surface, offset surface, Bezier surface, B-spline surface and coons patch.

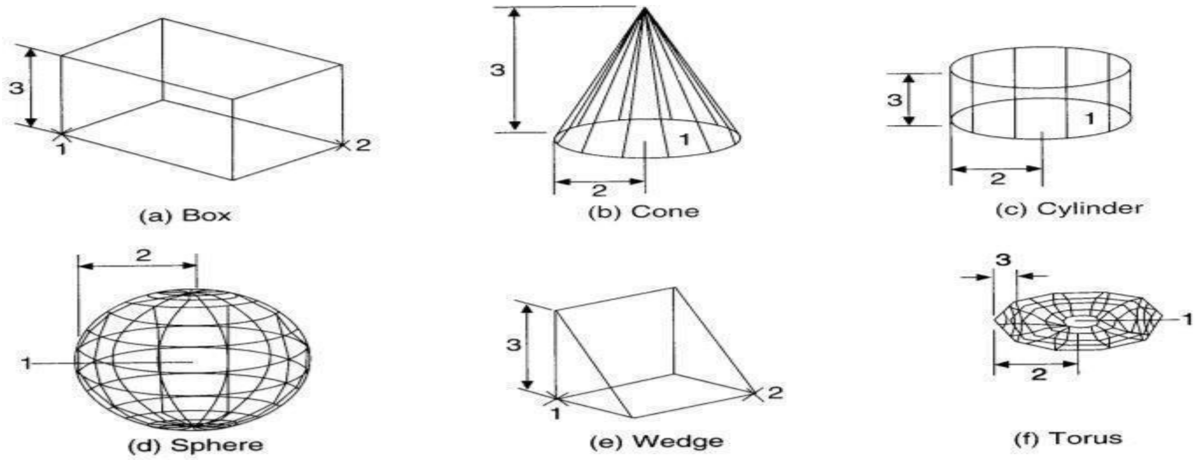


Surface entities.

Solid Modeling

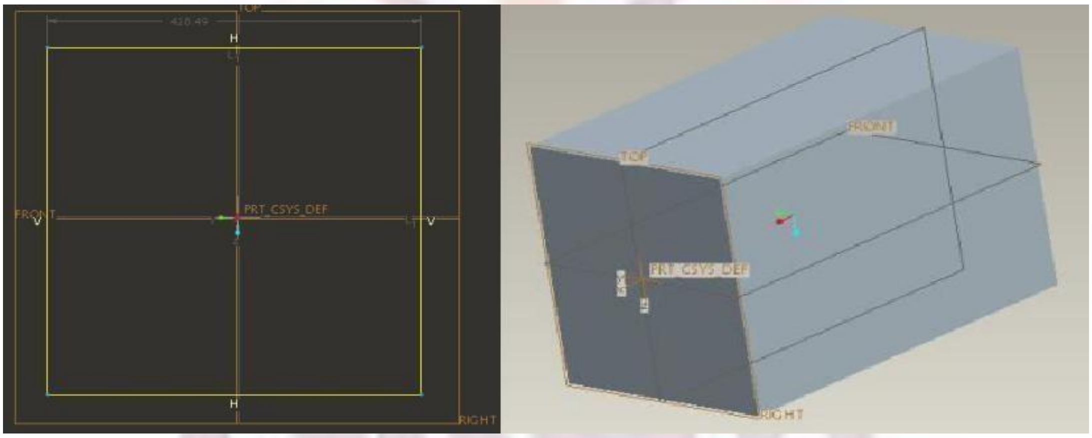
Solid models are constructed by the boundary representation (B-rep) method or by constructive solid geometry (CSG). The solid entities of CSG modeling are box, cone, sphere, cylinder, prism, wedge and torus. CSG modeling constructs solid models through Boolean operations (union, subtraction and intersection) on solid entities.

Extrusion



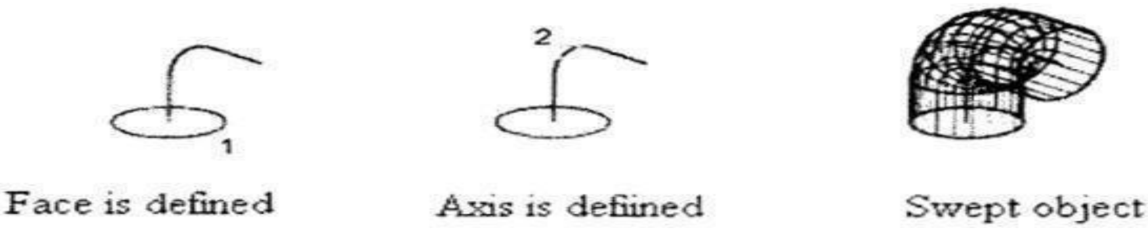
Solid primitives.

In extrusion, a three-dimensional solid is created by extruding the face in a direction perpendicular to it as shown in below figure.



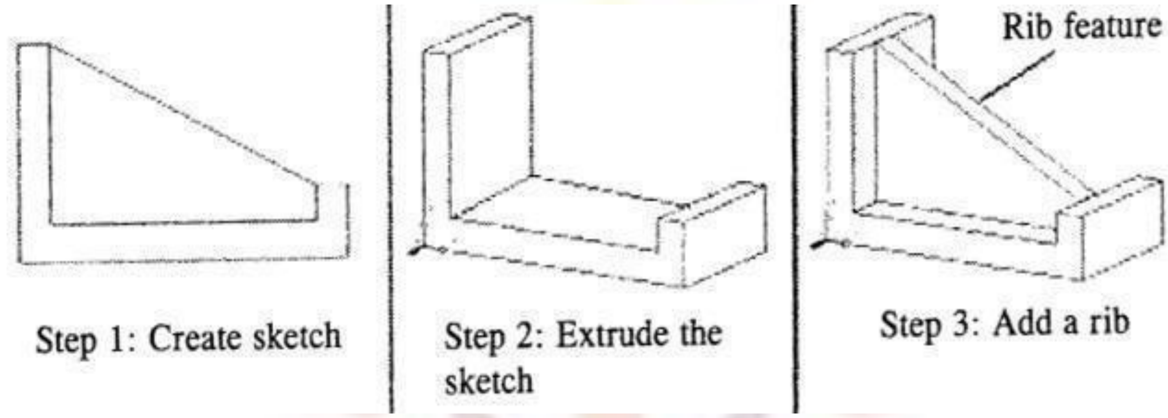
Sweeping

Sweeping is based on the notation of moving a point, curve, or a surface along a given path. A sweep may be linear or non-linear. The linear sweep may involve extrusion or revolving.



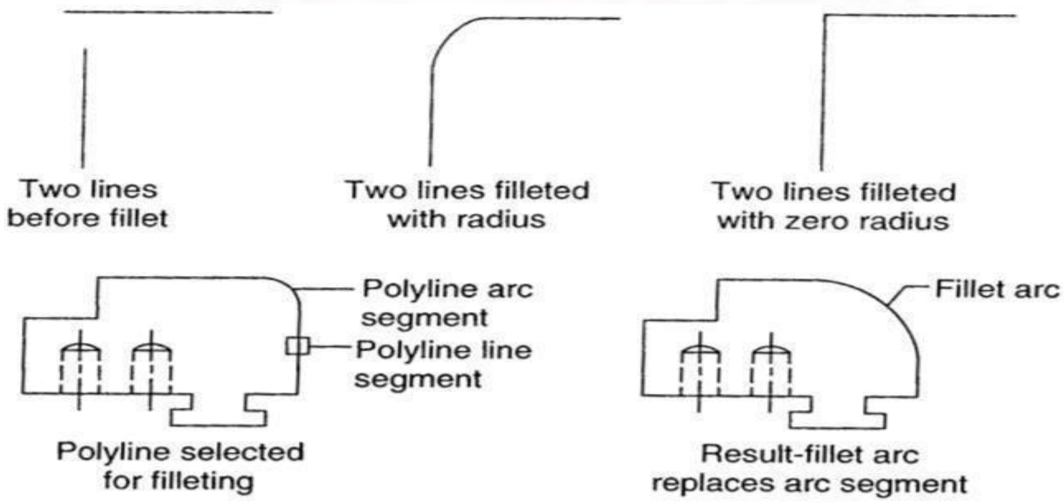
Feature Modeling

This creates solid models from a shape by an operation. The shape is a two- dimensional sketch for example, ribs, bosses, cuts and holes. The operation may involve extrusion, sweeping, revolving etc. Feature modeling is shown in below Figure.



Filleting

Filleting implies the rounding of a corner to eliminate its sharpness. The fillet radius is the radius of the arc that connects filleted objects. Changing the fillet radius affects subsequent fillets. If you set the fillet radius to 0, filleted objects are trimmed or extended until they intersect, but no arc is created. Filleting is shown in below Figure.



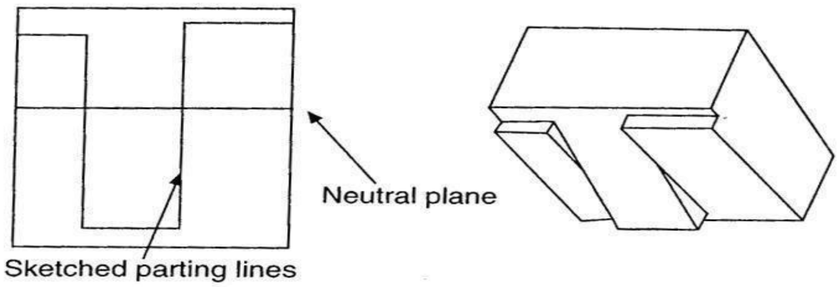
Filleting.

Tweaking

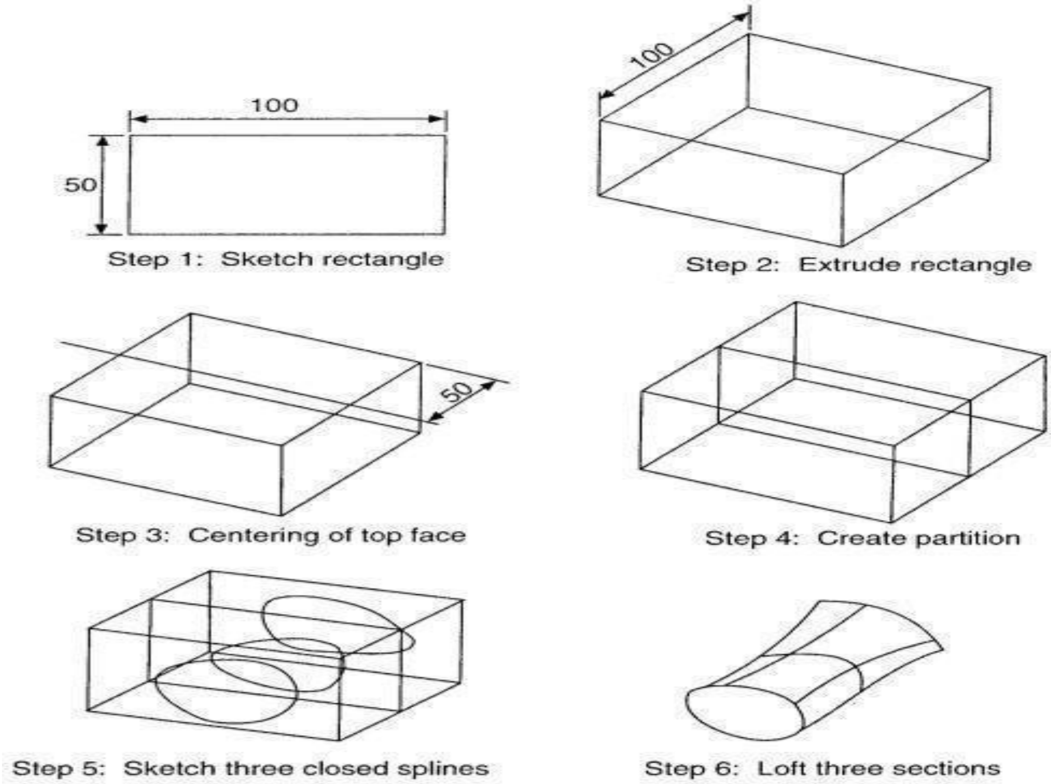
Tweaking uses several features to deform or alter (tweak) the surface of the part. Tweaking is not applicable to CSG solid models, splitting sketch drafts is shown in below Figure. This is because the CSG models retain the geometry and topology modeled from the primitives. The tweak menu lists the following options:

Lofting

1. Draft
2. Local Push
3. Radius Dome
4. Section Dome
5. Offset
6. Replace
7. Ear
8. Lip
9. Patch
10. Spinal Bend
11. Free Form
12. Draft Offset



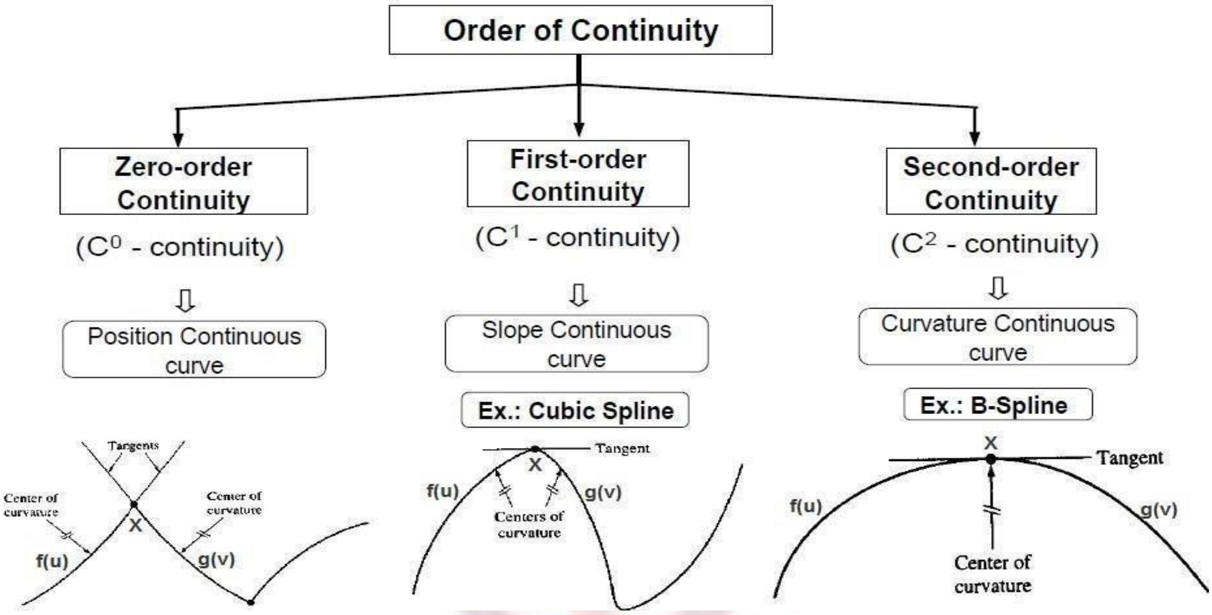
Lofting is used to create a model with a variant cross-section along a linear/non-linear axis. The lofting procedure is illustrated in below figure.



Lofting.

CURVATURE CONTINUITY

Mathematically, synthetic curves represented the problem of constructing a smooth curve that passes through given data points. Therefore the typical form of these curves is a polynomial. Various continuity requirements can be specified at the data points to impose various degrees of smoothness upon the resulting curve. The order of continuity becomes important when a complex curve is modeled by several curve segments pieced together end- to-end.



Two boundary curve segments shown in above figure are meeting at a vertex X. let these two curves be described as **f**(u) and **g**(v). Where u and v are values in intervals [a, b] and [m, n] respectively. The problem is: how these curves join together in a _smooth' way.

Consider the _endpoint' of curve **f**(u) and the _start point' of curve **g**(m). If **f**(b) and **g**(m) are equal as shown in above figure a, say curves **f**() and **g**() are C^0 continuity at **f**(b) = **g**(m).

If for all i<=k, the i-th derivatives at **f**(b) and **g**(m) are equal, say that the curves are C^k continuity at point **f**(b) = **g**(m). Intuitively,

- C^0 continuity (point/ position continuity) – Continuity of end point only (or) continuity of position
- C^1 continuity (tangent continuity) – Tangent continuity or first derivate if position
- C^2 continuity (curvature continuity) – Hydrodynamic character, light reflection curvaturecontinuity (or) second derivative of position

Bezier Curve

A Bezier curve is defined by a set of data points. The curve may interpolate or extrapolate the data points. Some CAD systems offer both options; others offer the interpolation version only. In both cases, the data points are used to control the shape of the resulting curves.

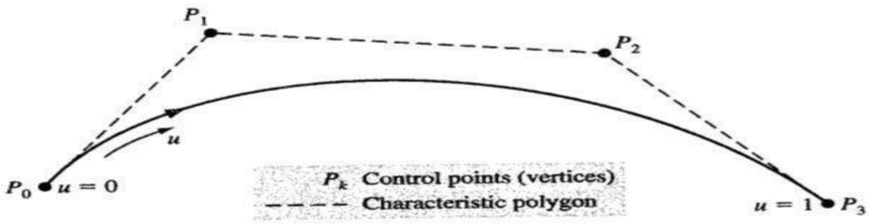
Bezier curves and surfaces are credited to P. Bezier of the French car firm Regie Renault, who developed (about 1962) and used them in his software system called UNISURF, which designers used to define the outer panels of several Renault cars. These curves, known as Bezier curves, were also independently developed by P. DeCasteljau of the French car company Citroen (about 1959), which used them as part of its CAD system. The Bezier UNISURF system was soon published in the literature; this is the reason that the curves now bear Beziers name.

As its mathematics show, the major characteristics of the Bezier curve are:

1. The shape of the Bezier curve is controlled by its defining points. Tangent vectors are not used in the curve development as is the case with the cubic spline. This allows the designer a much better feel for the relationship between input (points) and output (curve).
2. The order or the degree of Bezier curve is variable and is related to the number of points defining it. ***n*** + 1 points define an n^{th} degree curve, which permits higher-order continuity. This is not the case for cubic splines, where the degree is always cubic for a spline segment.

The data points of a Bezier curve are called control points. They form the vertices of what is called the control or characteristic polygon, which uniquely defines the curve shape as shown in Figure 6.30. Only the first and the last control points or vertices of the polygon actually lie on the curve. The other vertices define the order, derivatives, and shape of the curve. The curve is also always tangent to the first and last polygon segments. In addition, thecurve shape tends to follow the polygon shape.

These three observations should enable the user to sketch or predict the curve shape once its control points are given as illustrated in below Figure. The figure shows that the order of defining the control points changes the polygon definition, which changes the resulting curve shape. The arrow shown on each curve shows its parameterization direction.



Mathematically, for n + 1 control points, the Bezier curve is defined by the followingpolynomial of degree n:

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{P}_i B_{i,n}(u), \qquad 0 \le u \le 1 \qquad \longrightarrow 1$$

Where P(u) is a point on the curve and P_j is a control point. $B_{i,\ n}$ is the Bernstein polynomials. Thus, the Bezier curve has a Bernstein basis. The Bernstein polynomial serves as the blending or basis function for the Bezier curve and is given by

$$B_{i,n}(u) = C(n, i)u^i(1-u)^{n-i} \qquad \longrightarrow 2$$

Where C(n, i) is the binomial coefficient

$$C(n, i) = \frac{n!}{i!(n-i)!} \qquad \longrightarrow 3$$

Utilizing above two Equations 2 and 3 and observing that C(n, 0) = C(n, n) = 1, Equation 1can be expanded to give:

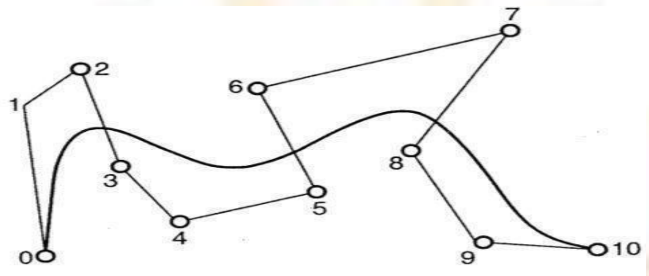
$$\mathbf{P}(u) = \mathbf{P}_0(1-u)^n + \mathbf{P}_1 C(n, 1)u(1-u)^{n-1} + \mathbf{P}_2 C(n, 2)u^2(1-u)^{n-2} + \cdots + \mathbf{P}_{n-1} C(n, n-1)u^{n-1}(1-u) + \mathbf{P}_n u^n, \qquad 0 \le u \le 1 \qquad \longrightarrow 4$$

Following are the important **characteristics** of a **Bezier curve**:

1. ***The degree of a Bezier curve defined by n + 1 control points is n:*** In each basis function, the exponent of u is i + (n - i) ~ n. Therefore, the degree of the curve is n.
2. ***P (u) passes through P_0 and P_n:*** The curve passes though the first and the last controlpoints.
3. ***Non-negativity:*** All basis functions are non-negative.
4. ***Bezier curves are tangent to their first and last legs***
5. ***Partition of unity:*** The sum of the basic functions at a fixed u is 1. It is not difficult to verify that the basic functions are the coefficients in the binomial expansion of the expression 1 = [u + (1 - u)] ^{n}. Hence, their sum is one. Moreover, since they are non-negative, we conclude that the value of any basis function is in the range of 0 and 1.

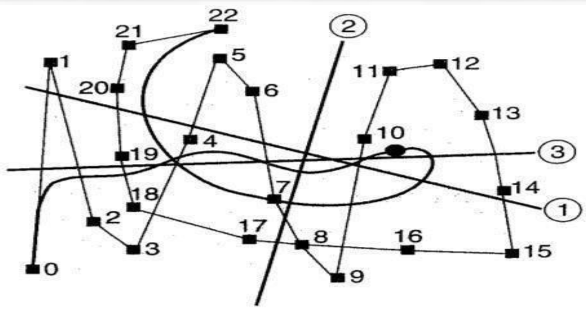
Since all basis functions are in the range of 0 and 1 and add to one, they can be considered as weights in the computation of a weighted average. More precisely, we could say "to compute **P**(u), one takes the weight $B_{n,\ i}$ (u) for control point $\mathbf{P_i}$ and sum them together."

6. ***Convex hull property:*** This means that the Bezier curve defined by the given n + 1 control points lays completely in the convex hull of the given control points. The convex hull of a set of points is the smallest convex set that contains all points. In below Figure, the convex hull of the 11 control points is shown. Note that not all control points are on the boundary of the convex hull. For example, control points 3, 4, 5, 8 and 9 are in the interior. The curve, except for the first two end points, lies completely in the convex hull. This property is important because we are guaranteed that the generated curve will be in an understood and computable region and will not go outside of it.



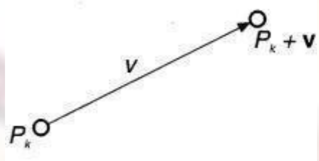
Illusration of convex hull property

7. ***Variation diminishing property:*** If the curve is in a plane, this means that no straight line intersects a Bezier curve more times than it intersects the characteristics polygon. Line 1 intersects the curve three times and the polyline seven times; line 2 intersects the curve and its polyline twice; the line 3-intersects the curve four times and the polyline seven times. You can draw other straight lines to verify this property. So, what is the meaning of this characteristic? It tells us that the complexity (i.e., turning and twisting) of the curve is no more complex than the characteristic polygon. In other words, the characteristic polygon twists and turns more frequently than the Bezier curve does, because an arbitrary line hits the control polyline more often than it hits the curve.

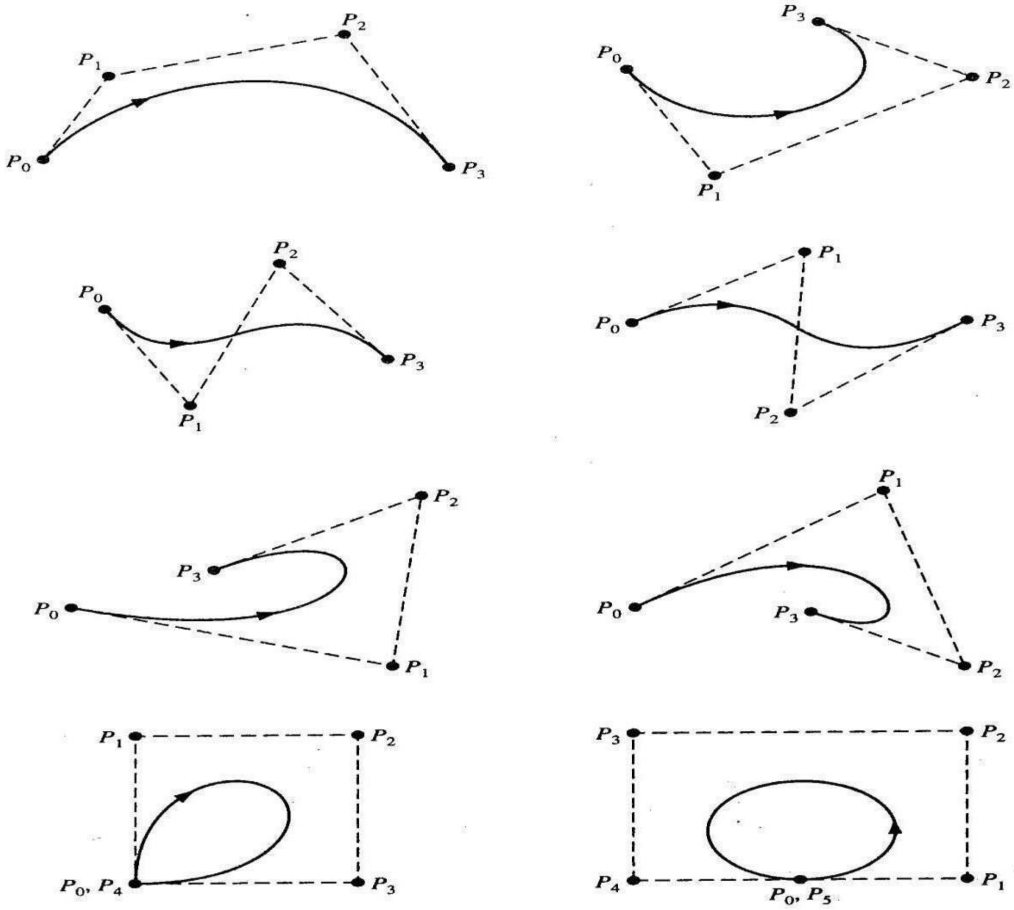


Complexity of characteristic polygon.

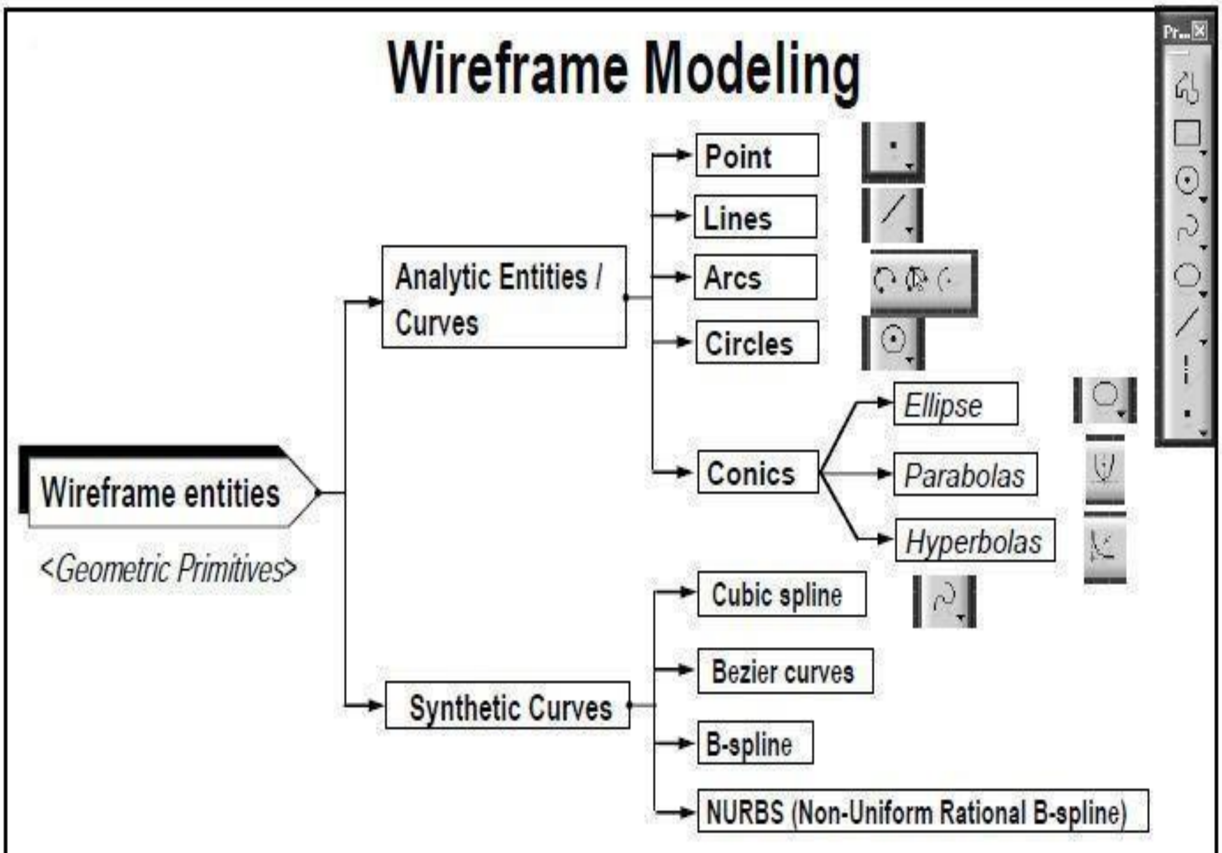
8. ***Affine invariance:*** If an affine transformation is applied to a Bezier curve, the result can be constructed from the affine images of its control points. This is a nice property. When we want to apply a geometric or even affine transformation to a Bezier curve, this property states that we can apply the transformation to control points, which is quite easy, and once the transformed control points are obtained the transformed Bezier curve is the one defined by these new points. Therefore, we do not have to transform the curve.
9. ***Moving control points:*** Changing the position of a control point will change the shape of the defined Bezier curve. Suppose a control point P_k is moved to a new position $\boldsymbol{P_k + v}$, where vector **v** gives both the direction and length of this move.



Effect of control point.

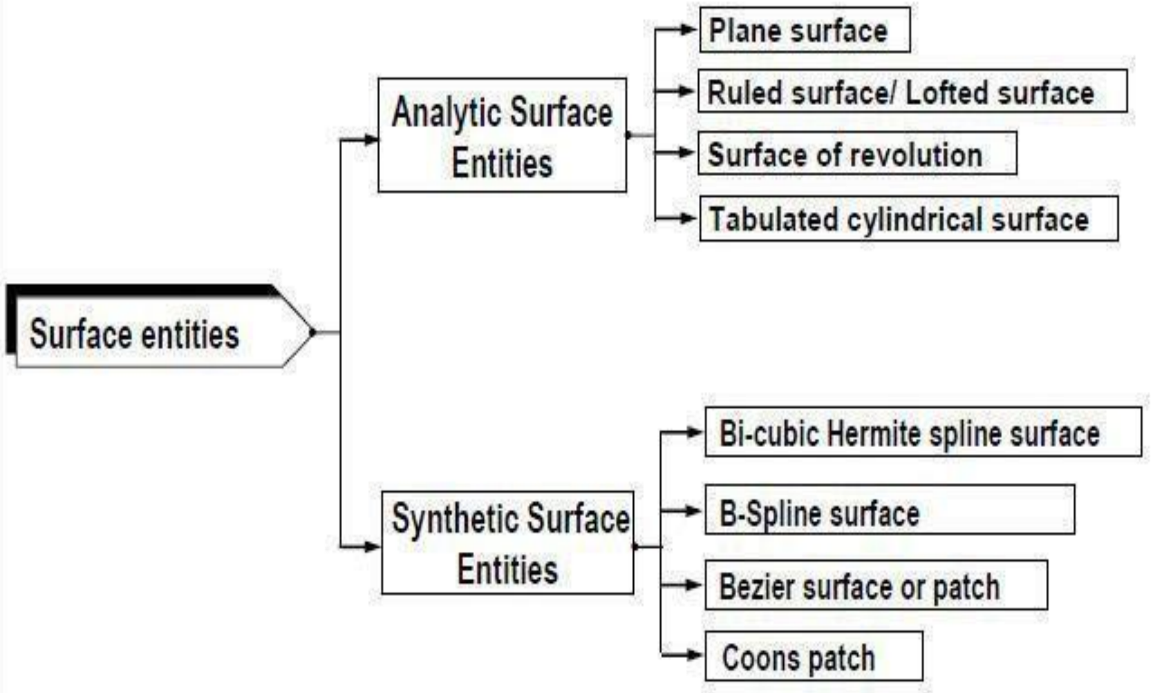


Cubic Bezier curve for various control points.



Analytic Curve	*Synthetic Curve*
- are described by analytic equations	- are described by a set of data points (i.e. control points)

Surface Modeling



Curve segment : is the fundamental building block for curve entities

Surface patch : is the fundamental building block for surfaces