ConceptlearningasSearch

- Conceptlearningcanbeviewedasthetaskofsearchingthroughalargespaceof hypotheses implicitlydefined bythe hypothesis representation.
- Thegoalofthissearchistofindthehypothesisthatbestfitsthetraining examples.

Example,theinstances***X***andhypotheses***H***inthe***EnjoySport***learningtask.The attribute***Sky***hasthreepossiblevalues,and***AirTemp,Humidity ,Wind,WaterForecast***eachhavetwopossiblevalues,theinstancespaceXcontains exactly

- 3.2.2.2.2.2=96Distinctinstances
- 5.4.4.4.4.4=5120syntacticallydistincthypotheseswithinH.

Everyhypothesiscontainingoneormore"Φ"symbolsrepresentstheemptysetof instances;thatis,itclassifieseveryinstanceas***negative.***

- 1+(4.3.3.3.3.3)=973. Semantically distinct hypotheses

General-to-SpecificOrderingofHypotheses

- Considerthetwohypotheses

  h_1=(Sunny,?,?,Strong,?,?)

  h_2=(Sunny,?,?,?,?,?)

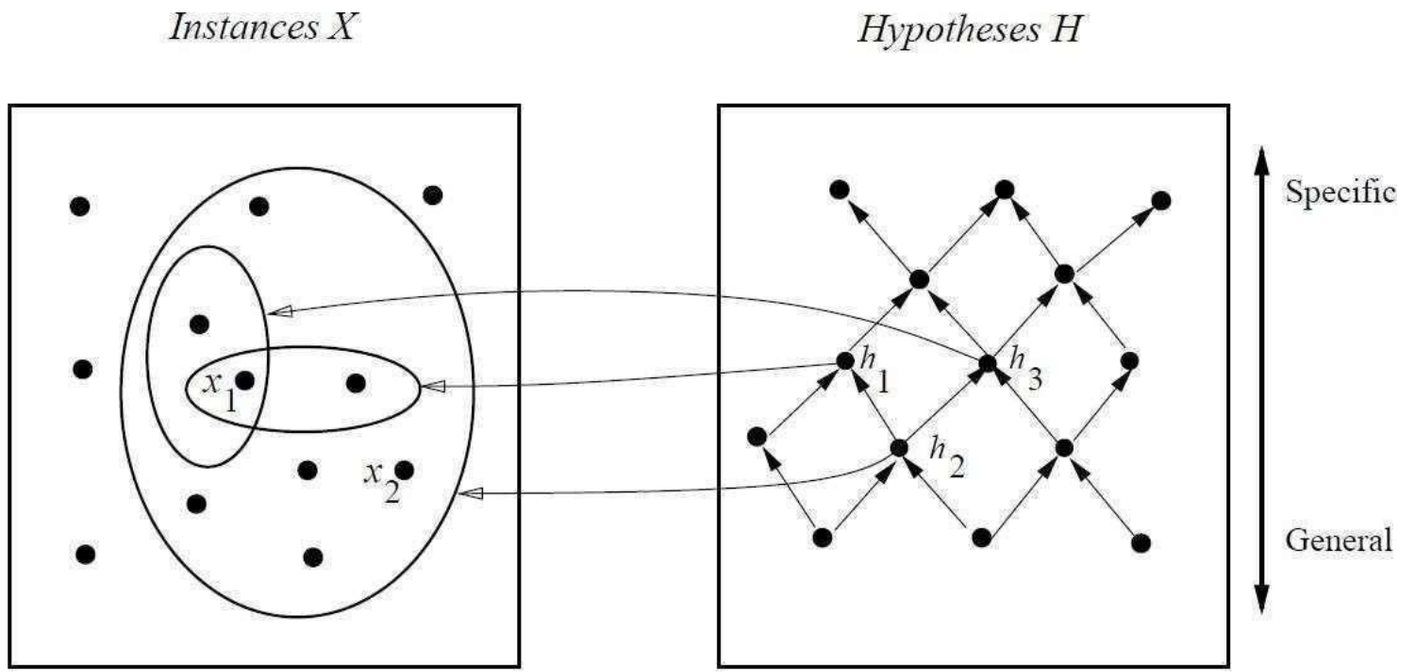
- Considerthesetsofinstancesthatareclassifiedpositivebyh$_1$andbyh_2.
- $h_2$imposesfewerconstraintsontheinstance, itclassifies more instancesaspositive. So,anyinstanceclassifiedpositivebyh$_1$ willalsobeclassifiedpositiveby h_2.Therefore,$h_2$ismoregeneralthanh$_1$.

General-to-SpecificOrderingofHypotheses

- Given hypotheses h_j and h_k, h_j is more-general-than or-equal do h_k if and only if any instance that satisfies h_k also satisfies h_i

Definition: Let h_j and h_k be Boolean-valued functions defined over X. Then h_j is more general-than-or-equal-to h_k (written $h_j \geq h_k$) if and only if

$$(\forall x \in X)[(h_k(x) = 1) \rightarrow (h_j(x) = 1)]$$



x_1 = <Sunny, Warm, High, Strong, Cool, Same>
x_2 = <Sunny, Warm, High, Light, Warm, Same>

h_1 = <Sunny, ?, ?, Strong, ?, ?>
h_2 = <Sunny, ?, ?, ?, ?, ?>
h_3 = <Sunny, ?, ?, ?, Cool, ?>

- Inthe figure, thebox on the leftrepresentsthesetXofallinstances, the box onthe rightthesetH ofall hypotheses.
- Eachhypothesiscorrespondstosomesubsetof X-thesubsetofinstancesthatitclassifies positive.
- The arrows connecting hypothesesrepresent the ***more - general - than***relation,withthearrowpointingtowardthelessgeneralhypothesis.
- Notethesubsetofinstancescharacterizedby$h_2$subsumesthesubset characterizedbyh_1,hence$h_2$ismore-general–thanh_1

FIND-S:FindingaMaximallySpecific Hypothesis

FIND-SAlgorithm

1. Initialize*h*tothemostspecifichypothesisin*H*
2. Foreachpositivetraininginstance*x*

   Foreachattributeconstraint$a_i$in*h*

   Iftheconstraint$a_i$issatisfiedby*x*

   Thendonothing

   Elsereplace$a_i$in*h*bythenextmoregeneralconstraintthatissatisfiedby*x*
3. Outputhypothesis*h*

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

ThefirststepofFIND-Sistoinitialize***h***tothemostspecifichypothesisin***H***

h-(Ø,Ø,Ø,Ø,Ø,Ø)

x_1=<*SunnyWarmNormalStrongWarmSame*>,+

Observing the first training example, it is clear that our hypothesis is too specific. Inparticular, none of the "Ø" constraints in h are satisfied by this example, so each isreplacedbythe nextmoregeneral constraintthatfitsthe example

h_1=<SunnyWarmNormalStrongWarmSame>

Thishisstillveryspecific;itassertsthatallinstancesarenegativeexceptforthesinglepositive training example

x_2=<*Sunny,Warm,High,Strong,Warm,Same*>,+

The second training example forces the algorithm to further generalize h, this timesubstituting a "?' in place of any attribute value in h that is not satisfied by the newexample

h_2=<SunnyWarm?StrongWarmSame>

x3=<Rainy,Cold,High,Strong,Warm,Change>,-

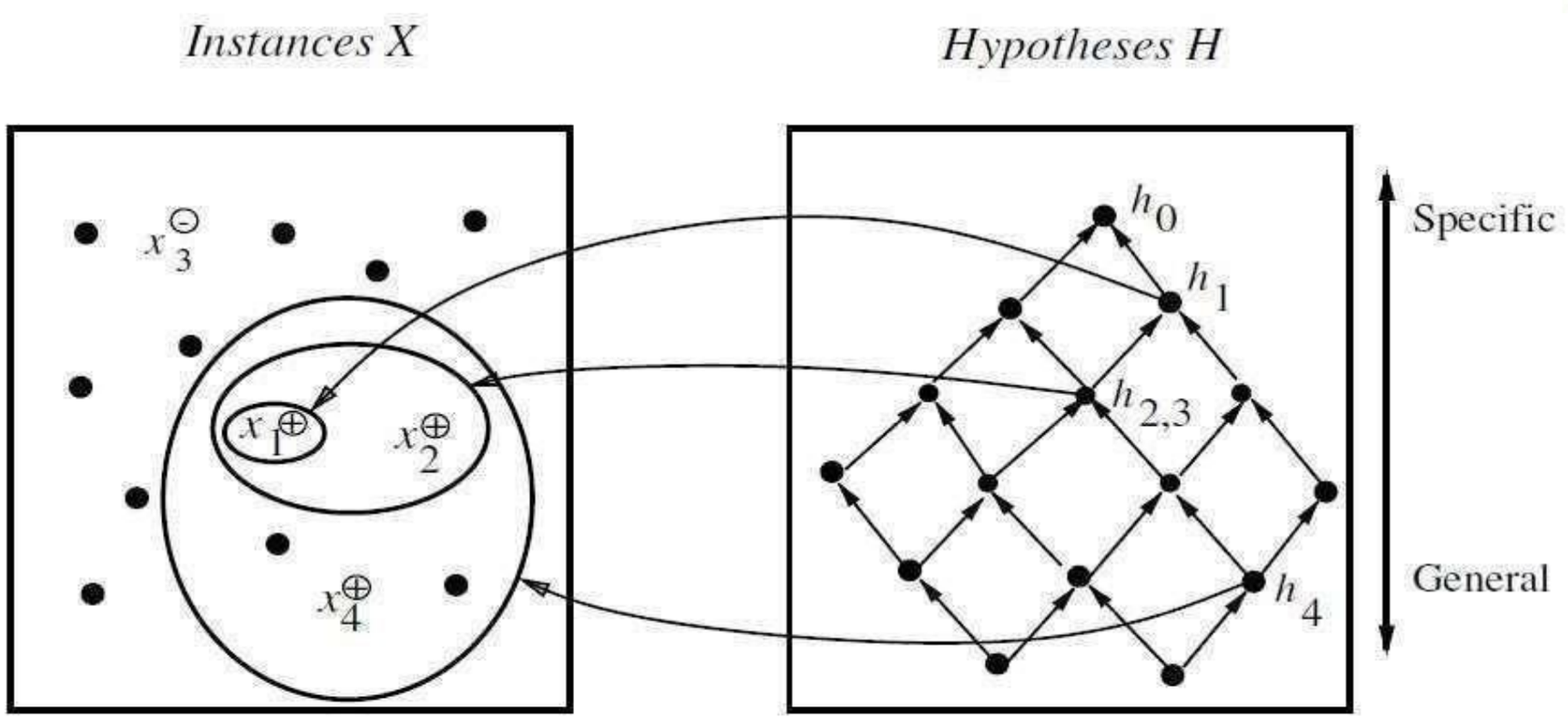
Uponencounteringthethirdtrainingthealgorithmmakesnochangetoh.TheFIND-Salgorithm simplyignores everynegative example.

h3=<SunnyWarm?StrongWarmSame>

x4=<SunnyWarmHighStrongCoolChange>,+

Thefourthexampleleadstoafurthergeneralizationofh

h4=<SunnyWarm?Strong??>



x_1 = <Sunny Warm Normal Strong Warm Same>, +

x_2 = <Sunny Warm High Strong Warm Same>, +

x_3 = <Rainy Cold High Strong Warm Change>, -

x_4 = <Sunny Warm High Strong Cool Change>, +

h_0 = <∅, ∅, ∅, ∅, ∅, ∅>

h_1 = <Sunny Warm Normal Strong Warm San

h_2 = <Sunny Warm ? Strong Warm Same>

h_3 = <Sunny Warm ? Strong Warm Same>

h_4 = <Sunny Warm ? Strong ? ? >

ThekeypropertyoftheFIND-Salgorithmis

- FIND-Sisguaranteedtooutputthemostspecifichypothesiswithin Hthat isconsistentwiththepositivetrainingexamples
- FIND-S algorithm's final hypothesis will also be consistent with the negativeexamples provided the correct target concept is contained in H, and provided thetrainingexamples are correct.

UnansweredbyFIND-S

1. Hasthelearnerconvergedtothecorrecttargetconcept?
2. Whypreferthemostspecifichypothesis?
3. Arethetrainingexamplesconsistent?
4. Whatiftherearesevеralmaximallyspecificconsistenthypotheses?

VersionSpaceandCANDIDATEELIMINATIONAlgorithm

ThekeyideaintheCANDIDATE-ELIMINATIONalgorithmistooutputadescriptionofthesetofall***hypothesesconsistentwiththetrainingexamples***

Representation

- ***Definition:***Ahypothesishis**consistent**with**a**setoftrainingexamples***D***if**and**onlyif***h(x)*** =***c(x)***foreachexample(x,***c(x))***in***D.***

$$Consistent(h,D) \equiv (\forall \langle x, c(x) \rangle \in D) h(x) = c(x))$$

Notedifferencebetweendefinitionsof*consistent*and*satisfies*

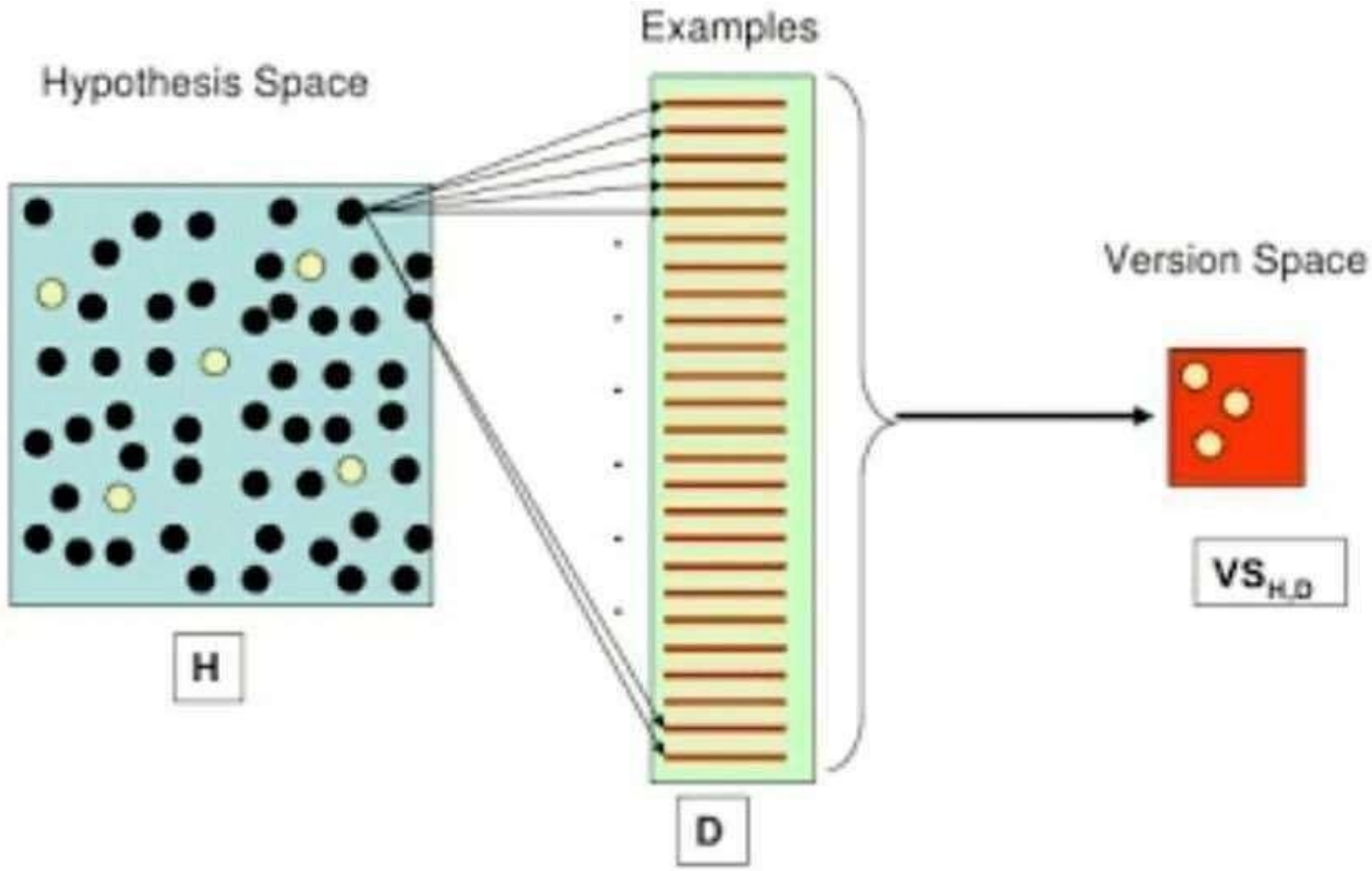
- Anexample*x*issaidto***satisfy***hypothesis***h***when***h(x)***=1,regardless ofwhether*x*isapositiveornegativeexampleofthetargetconcept.
- anexample*x*issaidto***consistent***withhypothesis***h***iff***h(x)=c(x)***

VersionSpace

ArepresentationofthesetofallhypotheseswhichareconsistentwithD

Definition: The **version space,** denoted $VS_{H,D}$with respect to hypothesisspace ***H***and training examples D, is the subset of hypotheses from ***H*** consistent with thetraining examplesin D

$$VS_{H,D} \equiv \{h \in H | Consistent(h,D)\}$$


Examples
Hypothesis Space
Version Space
H
D
VS H,D

TheLIST-THEN-ELIMINATE Algorithm

TheLIST-THEN-ELIMINATEalgorithmfirstinitializestheversionspacetocontainall hypotheses inH andthen eliminatesanyhypothesis foundinconsistentwith anytraining example.

TheLIST–THEN–ELIMINATE

Algorithm

1. VersionSpace *c* a listcontainingeveryhypothesisinH

2. Foreachtrainingexample, ***(x,c(x))***

   removefrom ***VersionSpace*** anyhypothesis ***h*** forwhich ***h(x)≠c(x)***

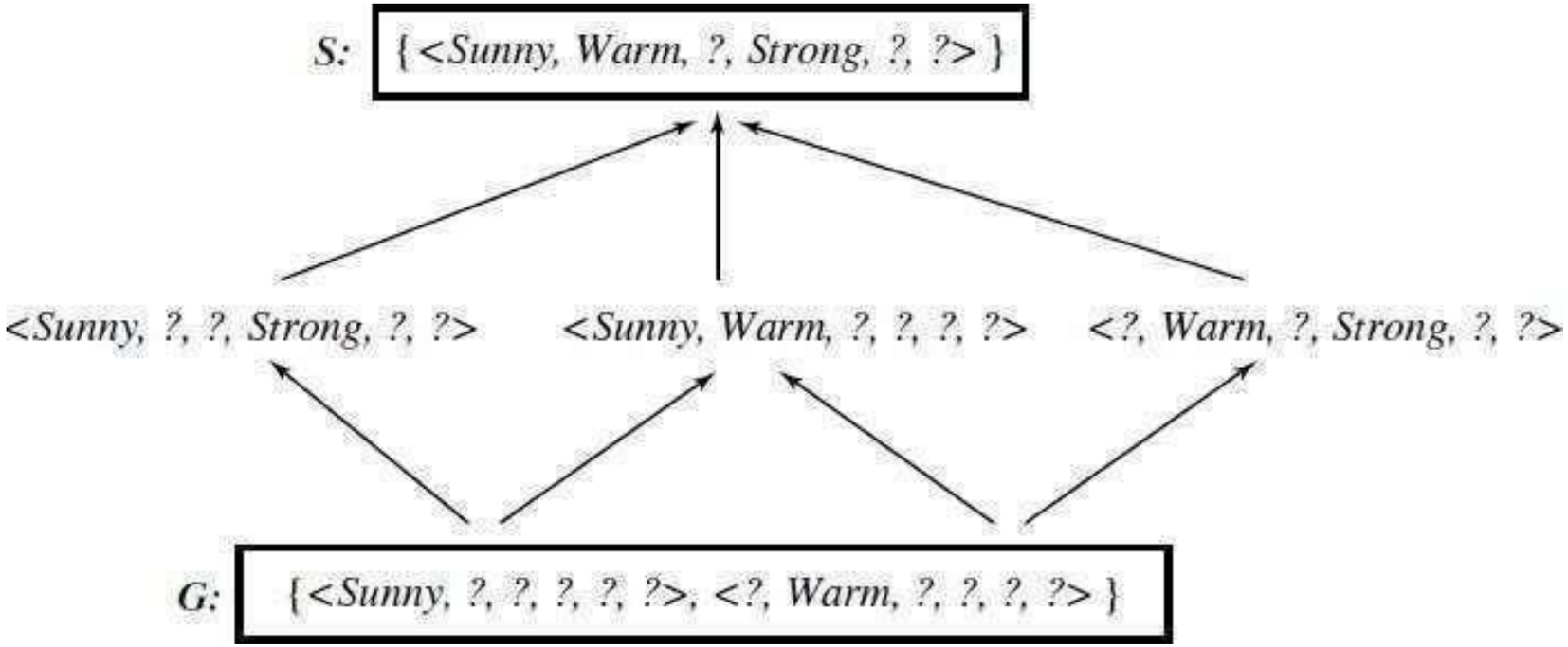
3. Outputthelistofhypothesesin ***VersionSpace***

TheLIST-THEN-ELIMINATEAlgorithm

- **List-Then-Eliminate** worksinprinciple,solongasversionspaceisfinite.
- However,sinceitrequiresexhaustiveenumerationofallhypothesesinpracticeitisnot feasible.

AMoreCompactRepresentationforVersionSpaces

- Theversionspaceisrepresentedbyitsmostgeneralandleastgeneralmembers.
- Thesemembersformgeneralandspecificboundarysetsthatdelimittheversionspace withinthepartiallyordered hypothesis space.



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4	Sunny	Warm	High	Strong	Cool	Change	Yes

- Aversionspacewithitsgeneral and specific boundarysets.
- The version space includes allsix hypotheses shown here, butcanberepresentedmoresimply by ***S***and***G.***
- Arrowsindicateinstanceof the*more-general-than* relation.Thisistheversion spaceforthe *Enjoysport*conceptlearning
- problemandtrainingexamples describedinbelowtable

$$G \equiv \{g \in H | Consistent(g, D) \wedge (\neg \exists g' \in H)[(g' >_g g) \wedge Consistent(g', D)]\}$$

Definition: The **specific boundary** S, with respect to hypothesis space *H* and training data *D*, is the set of minimally general (i.e., maximally specific) members of *H* consistent with *D*.

$$S \equiv \{s \in H | Consistent(s, D) \wedge (\neg \exists s' \in H)[(s >_g s') \wedge Consistent(s', D)]\}$$

VersionSpacerepresentationtheorem

Theorem: Let X be an arbitrary set of instances and Let H be a set ofBoolean-valued hypotheses defined over X. Let c : X →{O, 1} be an arbitrary target conceptdefined over X, and let D be an arbitrary set of training examples {(x, c(x))). For allX, H,c, and D such that Sand G are well defined,

$$VS_{H,D}=\{h\in H|(\exists s\in S)(\exists g\in G)(g\geq_g h\geq_g s)\}$$

$$VS_{H,D}=\{h\in H\mid(\exists s\in S)(\exists g\in G)(g\geq_g h \geq_g s)\}$$

ToProve:

1. Everyhsatisfyingtherighthandsideoftheaboveexpressionisin***VS*H,D**
2. Everymemberof***VS*H,D**satisfiestheright-handsideoftheexpression

Sketchofproof:

1. letg,h,sbearbitrarymembersofG,H,Srespectivelywith$g\geq_g h\geq_g s$

Bythe definitionof***S,s***mustbesatisfiedbyallpositiveexamplesinD.Because$h\geq_g s$,hmustalsobe satisfiedbyallpositiveexamplesinD.

Bythedefinitionof***G,***gcannotbesatisfiedbyanynegativeexampleinD,andbecause$g\geq_g h$hcannot be satisfied by any negative example in D. Because h is satisfied by all positive examples in Dandbynonegativeexamples inD,hisconsistentwithD,andthereforehisamemberof$VS_{H,D}$

2. Itcanbeprovenbyassumingsomehin$VS_{H,D}$,thatdoesnotsatisfytheright-handsideoftheexpression,thenshowingthatthisleadstoaninconsistency

TheCANDIDATE-ELIMINATIONLearningAlgorithm

The CANDIDATE-ELIMINTION algorithm computes the ***version space*** containingall hypotheses from H that are consistent with an observed sequence of trainingexamples.

Initialize G to the set of maximally general hypotheses in HInitialize S to the set of maximally specific hypotheses in HForeachtrainingexample d, do

- Ifdisapositiveexample
  - RemovefromGanyhypothesisinconsistentwithd
  - ForeachhypothesissinSthatisnotconsistentwithd
    - RemovesfromS
    - AddtoSallminimalgeneralizationshofssuchthat
      - hisconsistentwithd,andsomememberofGismoregeneralthanh
    - RemovefromSanyhypothesisthatismoregeneralthananotherhypothesisinS

- Ifdisanegativeexample
  - RemovefromSanyhypothesisinconsistentwithd
  - ForeachhypothesisginGthatisnotconsistentwithd
    - RemovegfromG
    - AddtoGallminimalspecializationshofgsuchthat
      - hisconsistentwithd,andsomememberofSismorespecificthanh
    - RemovefromGanyhypothesisthatislessgeneralthananotherhypothesisinG

AnIllustrativeExample

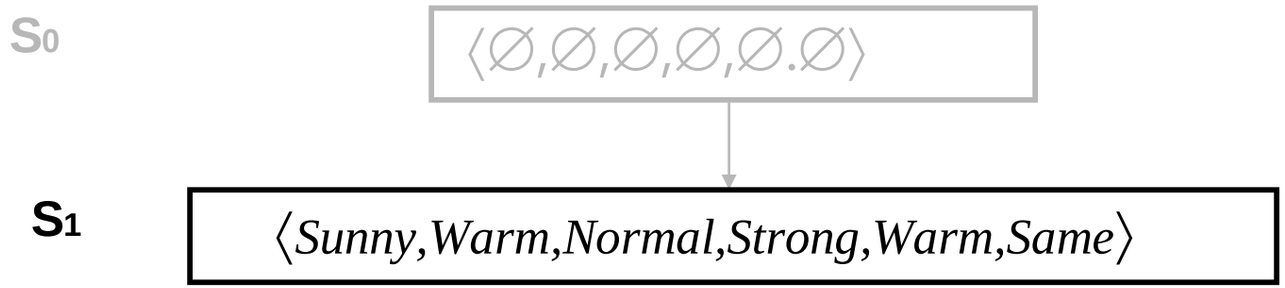
TheboundarysetsarefirstinitializedtoG_0andS_0,themostgeneralandmostspecifichypothesesinH.

S_0 ⟨∅,∅,∅,∅,∅,∅⟩

G_0 ⟨?,?,?,?,?,?⟩

Fortrainingexampled,

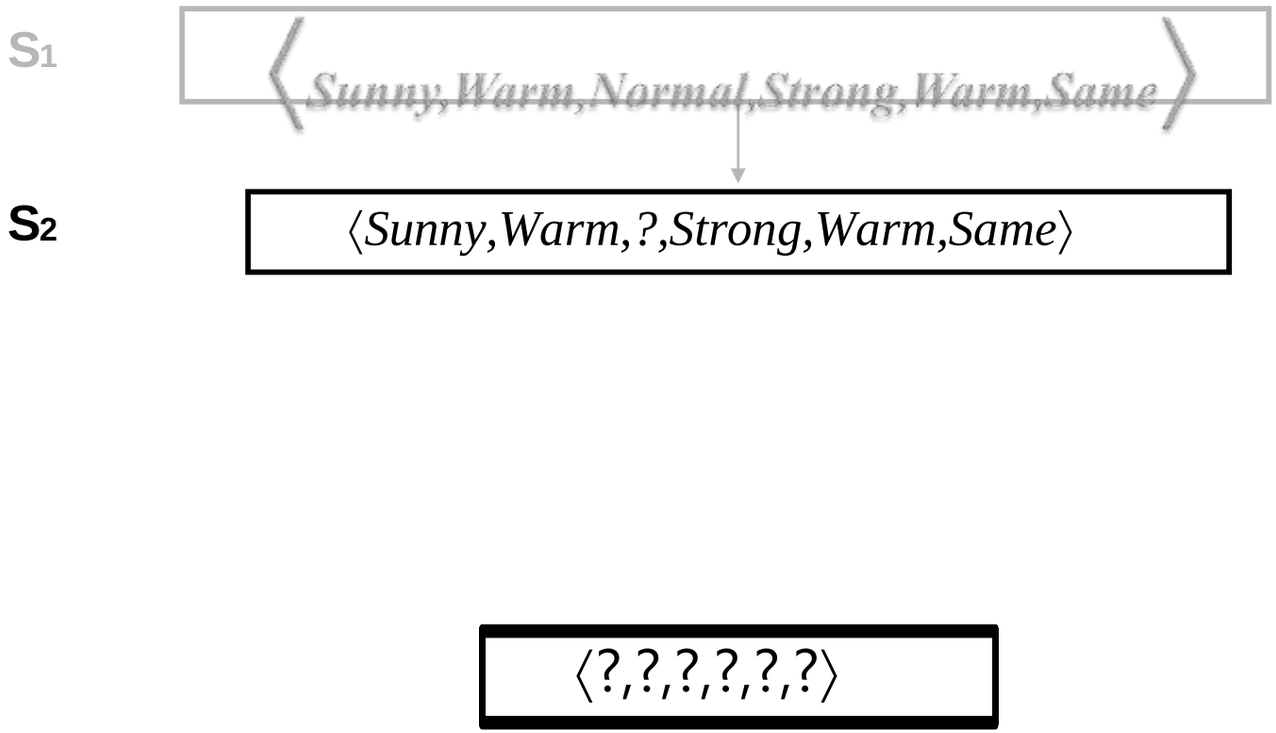
⟨Sunny,Warm,Normal,Strong,Warm,Same⟩+



⟨?,?,?,?,?,?⟩

Fortrainingexampled,

⟨Sunny,Warm,High,Strong,Warm,Same⟩+



Fortrainingexampled,

⟨Rainy,Cold,High,Strong,Warm,Change⟩–

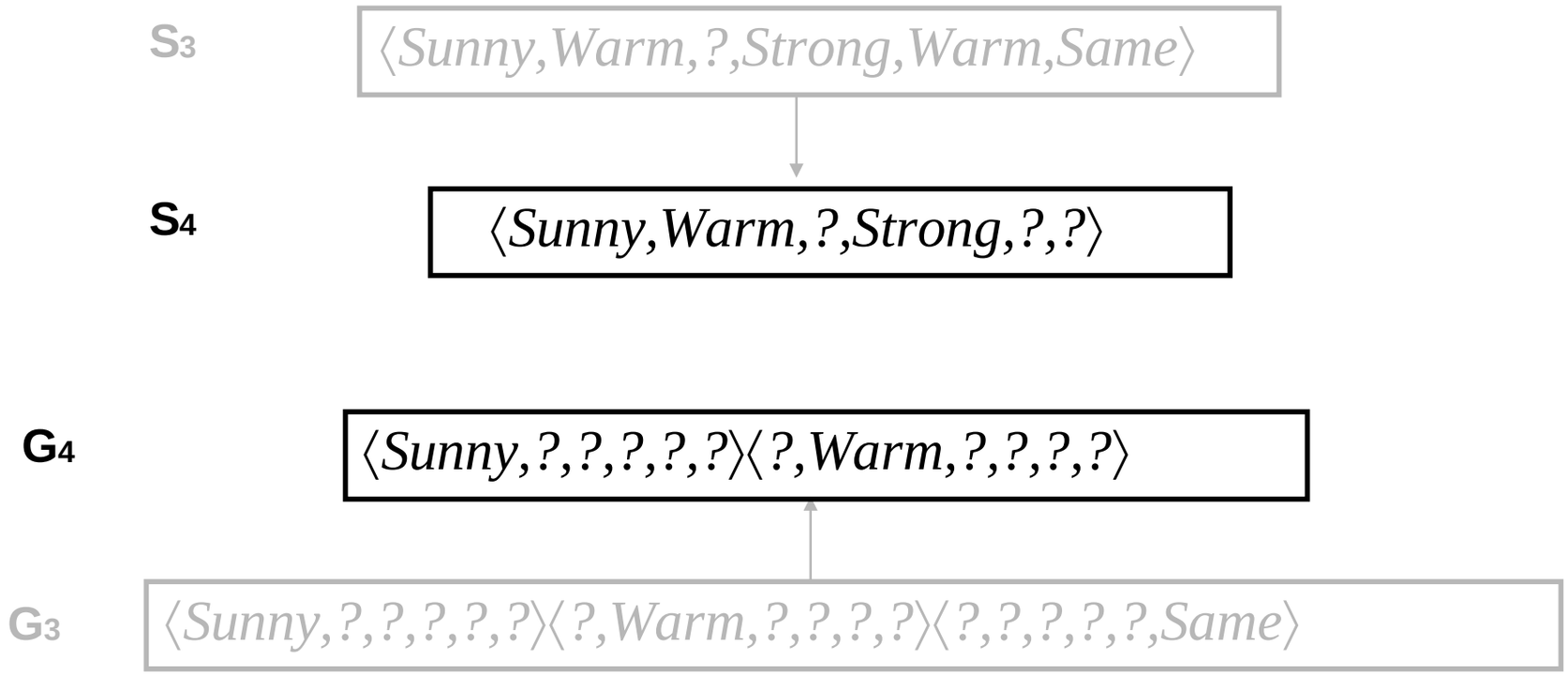
S_2, S_3 ⟨Sunny,Warm,?,Strong,Warm,Same⟩

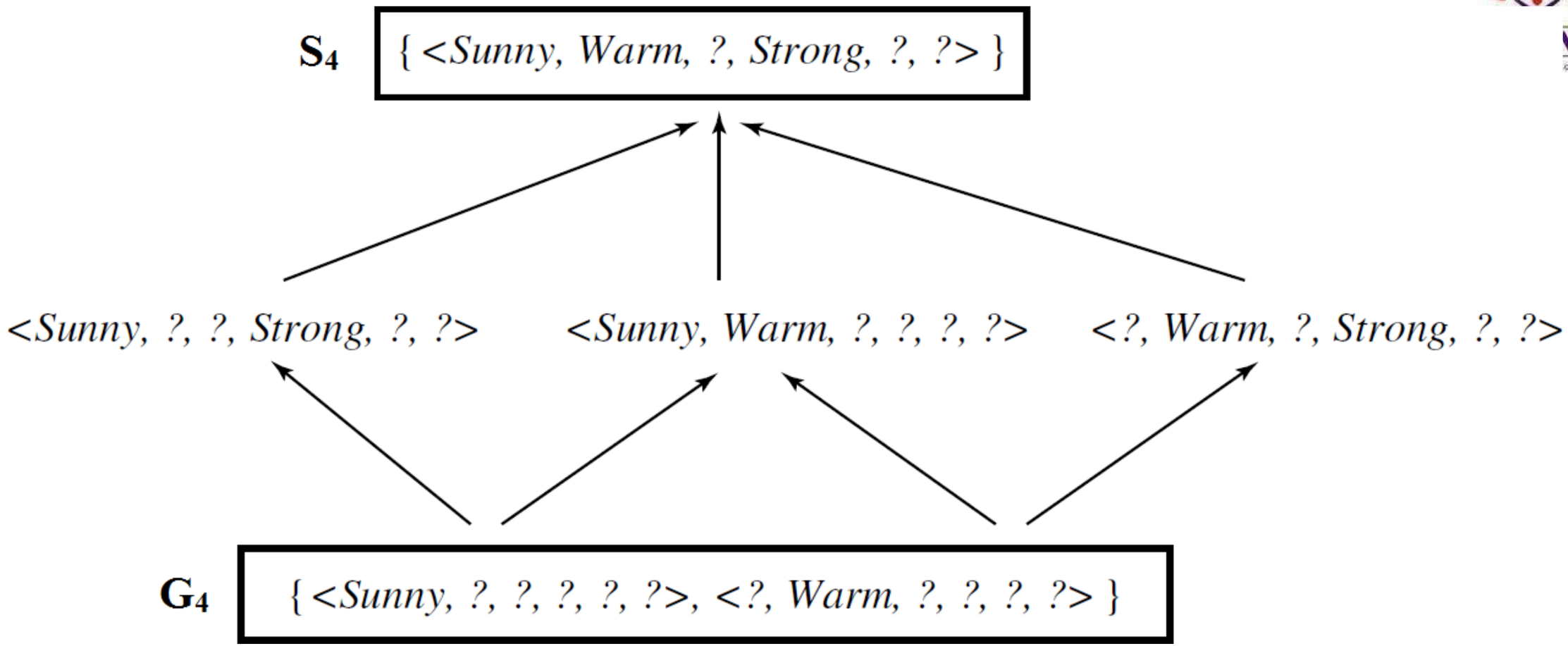
G_3 ⟨Sunny,?,?,?,?,?⟩⟨?,Warm,?,?,?,?⟩⟨?,?,?,?,?,Same⟩

G_2 ⟨?,?,?,?,?,?⟩

Fortrainingexampled,

⟨Sunny,Warm,High,Strong,CoolChange⟩+





Thefinalversionspaceforthe***EnjoySport***conceptlearningproblemand

InductiveBias

Thefundamentalquestionsforinductiveinference

- Whatifthetargetconceptisnotcontainedinthehypothesisspace?
- Canweavoidthisdifficultybyusing ahypothesisspacethatincludeseverypossiblehypothesis?
- Howdoesthesizeofthishypothesisspaceinfluencetheabilityofthealgorithmtogeneralizet ounobservedinstances?
- Howdoesthesizeofthehypothesisspaceinfluencethenumberoftrainingexamples thatmustbeobserved?

Effectofincompletehypothesisspace

Precedingalgorithmsworkiftargetfunctionis inH
Willgenerallynotworkiftargetfunctionnotin H

Considerfollowingexampleswhichrepresenttargetfunction
"sky=sunnyorsky=cloudy":

⟨SunnyWarmNormalStrongCoolChange⟩	Y
⟨CloudyWarmNormalStrongCoolChange⟩	Y
⟨RainyWarmNormalStrongCoolChange⟩	N

IfapplyCandidateEliminationalgorithmasbefore,endupwithemptyVersionSpaceAfterf

irsttwotrainingexample
S=⟨?WarmNormalStrongCoolChange⟩

Newhypothesisisoverlygeneralanditcoversthethirdnegativetrainingexample!Our

AnUnbiasedLearner

Incompletehypothesisspace

- IfcnotinH,thenconsidergeneralizingrepresentationofHtocontainc
- ThesizeoftheinstancespaceXofdaysdescribedbythesixavailableattributesis 96.Thenumberofdistinctsubsetsthatcanbedefinedoverasetxcontaining|X|elements(i.e., thesizeofthepowersetofX)is$2^{|X|}$
- Recallthatthereare96instancesin***EnjoySport***;hencethereare2^{96}possiblehypothesesinfullspaceH
- CandothisbyusingfullpropositionalcalculuswithAND,OR,NOT
- HenceHdefinedonlybyconjunctionsofattributesisbiased(containingonly973h’s)

- Let usreformulatethe ***Enjoysport***learningtaskin anunbiasedway by defininga newhypothesisspace***H'***thatcanrepresenteverysubsetofinstances;thatis,letH'correspondtoth epowersetofX.
- Onewayto definesuchan H'istoallowarbitrarydisjunctions,conjunctions,andnegationsofourearlierhypothes es.

Forinstance,thetargetconcept***"Sky*=*Sunny***or***Sky*=*Cloudy"***couldthenbedescribedas

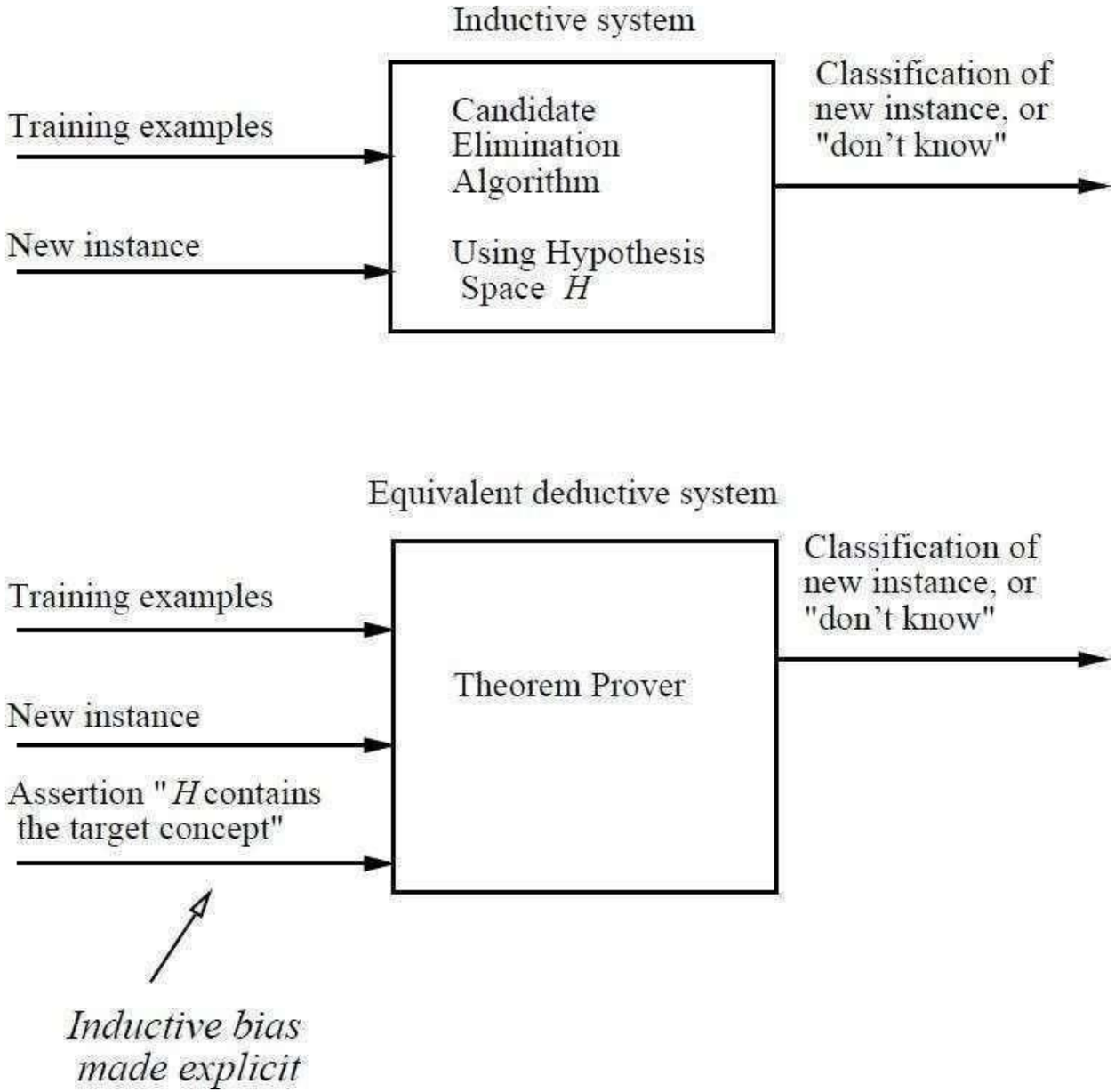
(Sunny,?,?,?,?,?)V***(Cloudy,***?,?,?,?,?)

Definition:

Consideraconceptlearningalgorithm L forthesetofinstances X.

- Letcbeanarbitraryconceptdefinedover X
- Let D_c={(x,c(x))}beanarbitrarysetoftrainingexamplesofc.
- Let $L(x_i, D_c)$ denotetheclassificationassignedtotheinstance x_i by L aftertrainingonthedata D_c.
- TheinductivebiasofLisany minimalsetofassertionsBsuch thatforanytargetconceptcandcorrespondingtrainingexamples D_c

$$(\forall \langle x_i \in X)[(B \wedge D_c \wedge x_i) \vdash L(x_i, D_c)]$$



Modelling inductive systems byequivalentdeductivesystems.

Theinput-outputbehavioroftheCANDIDATE-ELIMINATION algorithmusingahypothesisspace ***H*** is identical to that of a deductivetheoremproverutilizingthe assertion ***"H*** contains the target concept."Thisassertion is therefore called the***inductivebias***ofthe CANDIDATE-ELIMINATION algorithm.

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