A Course File On

"POWER SYSTEM OPERATION&CONTROL"

Submitted by

K.CHITANYA

Assistant Professor, EEE.

In the department of

Electrical & Electronics Engineering

NARASIMHA REDDY ENGINEERING COLLEGE

(Affiliated to J.N.T.U, HYDERABAD)

MAISAMMGUDA (V), DHULAPALLY (P), MEDCHAL (M)
SECUNDERABAD-14
(2025-2026)

INSTITUTE VISION AND MISSION

VISION:

To produce competent professionals who can contribute to the industry, research and societal benefits with environment consciousness and ethical values.

MISSION:

M1: Adapt continuous improvements in innovative teaching-learning practices and state-of-the-art infrastructure to transform students as competent professionals and entrepreneurs in multi-disciplinary fields.

M2: Develop an innovative ecosystem with strong involvement and participation of students and faculty members.

M3: Impart National development spirit among the students to utilize their knowledge and skills for societal benefits with ethical values.

DEPARTMENT VISION AND MISSION

MISSION:

To be recognized as a centre of excellence by nurturing competent and ethical electrical engineers, fostering innovation, advanced research and setting global benchmarks in developing sustainable solutions for societal progress.

MISSION:

Mission 1:

To impart high-quality technical education integrated with rigorous academic learning, while fostering innovation and creative problem-solving skills in the field of Electrical and Electronics Engineering.

Mission 2:

To promote a strong research culture among students and faculty to develop technologically advanced and cost-effective sustainable solutions for societal upliftment.

Mission 3:

To actively engage, collaborate, and build partnerships with leading industries, research and cademic institutions to strengthen technological advancements.

PROGRAM OUTCOMES (PO'S)

1	**Engineering Knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2	**Problem Analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3	**Design/Development of Solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4	**Conduct Investigations of Complex Problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5	**Modern Tool Usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6	**The Engineer and Society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7	**Environment and Sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8	**Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9	**Individual and Team Work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10	**Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and

	receive clear instructions.
11	**Project Management and Finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12	**Life-Long Learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSO’S)

PSO1 :(Engineering Knowledge and Analysis): Apply principles of engineering, electronics and computer science; physics, chemistry, environmental science, mathematics and laboratory skills for analyzing, testing, operation and maintenance of electrical systems, such as, electrical machines, power and energy systems etc

PSO2:(System Design): Ability to analyze, design, and realize physical systems, components or processes related to Electrical and Electronics Engineering systems applying the knowledge acquired from Network Theory, Electrical Machines, Power Systems, Control Systems, Power Electronics and other allied topics.

PSO3:(Application of the knowledge on society/environment): Apply the contextual knowledge of Electrical and Electronics Engineering to assess societal, environmental, health and safety with professional ethics and function effectively as an individual or a leader in a team to manage different projects in multidisciplinary environments as the process of life-long learning.

PROGRAM EDUCATIONAL OBJECTIVES(PEO'S)

(PEO'S)

PEO1 - Preparation: Graduates excel in higher education and become successful professionals in Electrical Engineering and its allied domains.

PEO2 -Core Competence: Graduates will have a strong foundation in engineering principles, scientific knowledge, and technical fundamentals, enabling them to address complex challenges in research, industry, higher education and entrepreneurship through rigorous learning.

PEO3- Professionalism & Lifelong Learning: Graduates shall have effective communication, teamwork, leadership, social responsibility, and ethical values while fostering adaptability and a commitment to lifelong learning.

Academic calendar

NARSIMHA REDDY ENGINEERING COLLEGE
UGC-AUTONOMOUS INSTITUTION
NRCM Your roots to success...

An Autonomous Institute
NAAC Accreditation 'A' Grade
Accredited by NBA
Approved by AICTE, Affiliated to JNTUH

ACADEMIC CALENDAR :: 2025-26
B.TECH IV YEAR I & II SEMESTER

I SEM

S.No.	Description	Duration		Duration (Weeks)
		From	To	
1	Commencement of I Semester class work	30.06.2025		
2	1st Spell of Instructions	30.06.2025	30.08.2025	9
3	First Mid Term Examinations	01.09.2025	06.09.2025	1
4	2nd Spell of Instructions (Including Dussera Recess)	08.09.2025	08.11.2025	9
5	Dussera Recess	29.09.2025	04.10.2025	1
6	Second Mid Term Examinations	10.11.2025	15.11.2025	1
7	End Semester Examinations	17.11.2025	29.11.2025	2
8	Lab Examinations	01.12.2025	06.12.2025	1

II SEM

S.No.	Description	Duration		Duration (Weeks)
		From	To	
1	Commencement of II Semester class work	08.12.2025		
2	1st Spell of Instructions	08.12.2025	07.02.2026	9
3	First Mid Term Examinations	09.02.2026	11.02.2026	3 Days
4	2nd Spell of Instructions	12.02.2026	08.04.2026	8
5	Second Mid Term Examinations	09.04.2026	11.04.2026	3 Days
6	End Semester Examinations	13.04.2026	18.04.2026	1
7	Project Work Viva–Voce	20.04.2026	25.04.2026	1

Copy to:
1. Deans
2. IQAC
3. All HODs
4. Administrative Officer
5. Account officer
6. Web Portal I/C
7. ERP I/C
8. Library
9. Student Notice Boards

PRINCIPAL
NARASIMHA REDDY ENGINEERING COLLEGE
UGC AUTONOMOUS
Survey No.518, Maisammaguda (V), Dhulapally (P)
Medchal (M), Medchal Dist., Hyderabad-500100

POWER SYSTEM OPERATION AND CONTROL

III Year B.Tech. II-Sem

Course Code	Category	Hours/ Week			Credits	Maximum Marks		
23EE602	Core	L	T	P	3	CIE	SEE	TOTAL
		3	0	0		40	60	100
Contact Classes: 48	Tutorial Classes: 0	Practical Classes: Nil				Total Classes:48		

Pre-requisites: Power System-I, Power System-II

Course Objectives:

- Understand the principles and significance of real power control, emphasizing the importance of frequency control in power systems.
- Analyze various methods for effective reactive power control in power systems.
- Grasp the concepts of unit commitment, economic load dispatch, and real-time control, highlighting their importance in power system operation.

Course Outcomes: At the end of the course the student will be able to:

- Understand operation and control of power systems.
- Analyze various functions of EMS functions and stability of machines.
- Understand power system deregulation and restructuring
- Analyze whether the machine is in stable or unstable position.
- Test the stability of the power system.

UNIT I

Load Flow Studies: Introduction, Bus classification -Nodal admittance matrix - Load flow equations - Iterative methods - Gauss and Gauss Seidel Methods, Newton-Raphson Method-Fast Decoupled Method-Merits and demerits of the above methods-System data for load flow study

UNIT II

Economic Operation Of Power Systems: Distribution of load between units within a plant-Transmission loss as a function of plant generation, Calculation of loss coefficients-Distribution of load between plants.

UNIT III

PF Control: Introduction, load frequency problem-Megawatt frequency (or P-f) control channel, MVAR voltages (or Q-V) control channel-Dynamic interaction between P-f and Q-V loops. Mathematical model of speed- governing system-Turbine models, division of power system into control areas, P-f control of single control area (the uncontrolled and controlled cases)-P-f control of two area systems (the uncontrolled cases and controlled cases).

UNIT IV Power System Stability: The stability problem-Steady state stability, transient stability and Dynamic Stability-Swing equation. Equal area criterion of stability-Applications of Equal area criterion, Step by step solution of swing equation-Factors affecting transient stability, Methods to improve steady state and Transient stability, Introduction to voltage stability

UNIT-V Computer Control of Power Systems: Need of computer control of power systems. Concept of energy control centre (or) load dispatch centre and the functions - system monitoring - data acquisition and control. System hardware configuration – SCADA and EMS functions. Network topology – Importance of Load Forecasting and simple techniques of forecasting

TEXT BOOKS:

1. C. L. Wadhwa, Electrical Power Systems, 3rd Edn, New Age International Publishing Co., 2001.
2. D. P. Kothari and I. J. Nagrath, Modern Power System Analysis, 4th Edn, Tata McGraw Hill Education Private Limited 2011.

REFERENCE BOOKS:

1. D. P. Kothari: Modern Power System Analysis-Tata Mc Graw Hill Pub. Co. 2003.
   Hadi Sadat: Power System Analysis –Tata Mc Graw Hill

CO-PO MAPPING

Attributes	Knowledge	Analysis	Design	Develop	Modern Tools	Society	Environment	Ethics	Team Work	Communication	Project Management Finance	Life long Learning
Course Outcome	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PO8**	**PO9**	**PO10**	**PO11**	**PO12**
CO1	3	3	1	1	-	-	-	-	-	-	-	-
CO2	3	2	1	-	-	-	-	-	-	-	-	-
CO3	3	3	3	1	-	-	-	-	-	-	-	-
CO4	3	2	3	-	-	-	-	-	-	-	-	-
CO5	3	2	3	1	1	-	-	-	-	-	-	-
Avg	**3**	**2.2**	**2**	**1**	**1**	-	-	-	-	-	-	-

MAPPING OF COURSE OUTCOMES WITH PSO's

COs	PSO1	PSO2	PSO3
CO1	3	3	-
CO2	3	2	-
CO3	3	3	-
CO4	2	2	-
CO5	3	3	1
Avg	**2.8**	**2.6**	**1**

Nominal Rolls

S.No	Roll Number	Full Name
1	23X01A0201	ANDE JEEVAN REDDY
2	23X01A0202	BAKKA PRAVEEN KUMAR
3	23X01A0204	BATIINENI ABHIRAM
4	23X01A0205	BONTHA JAGADISH
5	23X01A0206	CHELLA KIRAN KUMAR
6	23X01A0207	CHIKKUDU ESHWAR
7	23X01A0208	CHILIVERU VIJAY KUMAR
8	23X01A0209	CHITEMPALLY THARUN
9	23X01A0210	DHARAVATH AKHIL
10	23X01A0211	EMMADI HAR!
11	23X01A0212	JALLI CHAITANYA
12	23X01A0214	KOMMETA RAMDAS
13	23X01A0215	KUNCHAM PRANITH
14	23X01A0216	MADIREDDY DEEKSHITHA REDDY
15	23X01A0219	MORALA SAI KIRAN
16	23X01A0221	NAYENI ARAVIND
17	23X01A0222	PICHUKULA VARSHINI
18	23X01A0223	PILLY RAKESH
19	23X01A0224	PUTIALA UDAY KIRAN
20	23X01A0225	RAIPALLY PRANAY
21	23X01A0226	REDDY MANOHAR
22	23X01A0227	SUDAY
23	23X01A0228	SAMALA SIDDARTHA
24	23X01A0229	SIGINAM PRAGATH
25	23X01A0230	SONAPURAM PRAVEENKUMAR
26	23X01A0231	VASI SANDHYA RANI
27	24X05A0201	ALLE NITHISH KUMAR
28	24X05A0202	CHEKOLEKAR AKASH
29	24X05A0203	CHINNOLLA SHIVA RAJ
30	24X05A0204	JATOTHU SRIKANTH
31	24X05A0205	MALYALA NITHISH
32	24X05A0206	MOOD NIKHITHA
33	24X05A0207	RAJARAPU TEJA

Time table

NARSIMHA REDDY ENGINEERING COLLEGE
UGC AUTONOMOUS INSTITUTION
Maisammaguda (V), Kompally - 500100, Secunderabad, Telangana state, India

Accredited by NBA & NAAC with 'A' Grade
Approved by AICTE
Permanently affiliated to JNTUH

Department of Electrical and Electronics Engineering
TIME TABLE
AY (2025-2026)

YEAR: III YEAR II SEM
Class In-charge: Mr.V.Ramudu

ROOM NUMBER:MT 320
w.e.f : 15-12-2025

DAY/TIME	1	2	3		4	5	6
H/T	9.30AM-10.30AM	10.30AM-11.30AM	11.30AM-12.30PM	12.30 PM -1.20PM	1.20PM-2.10PM	2.10PM-3.00PM	3.00PM-3.50PM
Mon	DSP	PSD	PSP	LUNCH	PSOC	DSP	FIOT
Tue	PSOC	DSP	PSP		PS LAB		
Wed	FIOT	PSP	PSD		ES	PSOC	DSP
Thu	PSD	FIOT	ES		PSD	PSP	PSOC
Fri	DSP LAB				PSD	DSP	FIOT
Sat	FIOT	PSOC	PSP		CS LAB		

NR23

23EE601	Power System Protection	Mr.M.Nataraj
23EE602	Power System Operation and Control	Mr.K.Chitanya
23EC607	Digital Signal Processing	Mr.Sridhar Reddy
23EE603	Power Semiconductor Drives	Mr.V.Ramudu
23EC614	Fundamentals of Internet of Things	Mrs.M.Veena
23EE608	Power System Laboratory	Mr.V.Ramudu/ Mr.K.Chitanya
23EE609	Control Systems Laboratory	Mrs.A.Lakshmi devi/Mr.K.Chitanya
23EC610	Digital Signal Processing Lab	Mr.Sridhar Reddy
23EE611	Industry Oriented Mini Project / internship	Dr.C.Sasikala/Mr.V.Prashanth
*MC6001	Environmental Science	Ms.B.Rupa/ Mrs.A.Lakshmi devi

Timetable Incharge

HOD-EEE

HEAD OF THE DEPARTMENT
ELECTRICAL & ELECTRONICS ENGINEERING
NARASIMHA REDDY ENGINEERING COLLEGE
MAISAMMAGUDA, DHULLAPALLY(P)
SECUNDERABAD, TELANGANA

Principal

PRINCIPAL
NARSIMHA REDDY ENGINEERING COLLEGE
UGC AUTONOMOUS
Sy.No. 518, Maisammaguda (V), Dhulapally (P)
Medchal (M & Dist.), Hyderabad-500100. T.G

Individual Time table

FACULTY NAME: MrK.CHAITANYA IV YEAR CLASS INCHARGE

Day/Time	9:30 to 10:30	10:30 to 11:30	11:30 to 12:30	LUNCH	1:20 to 2:10	2:10 to 3:00	3:00 to 3:50
Mon					PSOC		
Tue	PSOC				PS LAB		
Wed	EM	PSOC			PSOC		
Thu	EM	PSOC					
Fri					EM		
Sat	EM		PSOC		CS Lab		

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LESSON PLAN

Branch: EEE Year: I I I Semester: II Section: A Academic Year: 2025-2026

Subject: POWERSYSTEM OPERATION&CONTROL

Name of the faculty: K.CHAITANYA

S. No.	Date (As per Academic calendar)	Topics to be covered	Remarks
1.	8/12/2025	**UNIT – I: LOAD FLOW STUDIESS**	
2.	8/12/2025	Introduction	
3.	9/12/2025	Types of buses	
4.	10/12/2025	Static load flow equations	
5.	13/12/25	Formation of ybus from load flow equations	
6.	15/12/2025	Formation of bus im pedance matrices	
7.	16/12/2025	Gauss seidel iterative method	
8.	17/12/2025	numericals	
9.	17/12/2025	Newton Raphson method,numericals	
10.	18/12/2025	Fast decoupled method	
11.	20/12/2025	derivation	
12.	22/12/2025	Problems, merits and de-merits of fast decoupled methods.	
13.	23/12/2025	**UNIT – II** **Economic operation of power systems**	
14.	24/12/2025	introduction	
15.	27/12/2025	Distribution of load between units within a plant	
16.	29/12/2025	Transmission loss as a function of plant generation	
17.	30/12/2025	Calculation of loss coefficients	
	31/12/2025	Numericals	
18	1-1-2026	**UNIT – III** **Pf control**	
19	3/1/2026	Introduction	
20	5-1-2026	, load frequency problem-Megawatt frequency (or P-f) control channel	
21	6/1/2026	MVAR voltages (or Q-V) control channel-Dynamic interaction between P-f and Q-V loops.	
22	7/1/2026	. Mathematical model of speed- governing system	
23	8/1/2026	Turbine models	

24	9/1/2026	division of power system into control areas	
25	10/1/2026	p-f control of single area system	
26	19/1/2026	p-f control of two area system	
27	20/1/2026	numericals	
28	21/1/2026	numericals	
29	22/1/2026	numericals	
30	24/1/2026	**UNIT-IV POWER SYSTEM STABILITY**	
31	27/1/2026	introduction	
32	28/1/2026	The stability problem	
33	29/1/2026	Steady state stability	
34	30/1/2026	Transient state stability	
35	31/1/2026	Dynamic stability	
36	2/2/2026	Swing equation	
37	3/2/2026	Equal area criteria	
38	4/2/2026	Applications of equal area criteria	
39	5/2/2026	Step-by-step solution of swing equation	
40	6/2/2026	Factors effecting on stability,numericals,voltage stability	
41	7/2/2026	**UNIT – V** **Computer control of power system**	
42	16/2/2026	introduction	
43	17/2/2026	Need of computer control of power systems	
44	21/2/2026	Importance of ecs system	
45	23/2/2026	Concept of load dispatch centre	
46	1/3/2026	Functions of EHS	
47	2/3/2026	system monitoring - data acquisition and control. System	
48	4/3/2026	hardware configuration – SCADA and EMS functions	
49	6/3/2026	Network topology – Importance of Load Forecasting and simple techniques of forecasting	
50	7/3/2026	Importance of Load Forecasting and simple techniques of forecasting	
51	8/3/2026	Numericals	

Course Instructor **Head of the Dept.** **Principal**

Optimizing System Efficiency

- **Minimizing losses:** Power systems inevitably experience losses due to resistance in transmission lines and equipment. Load flow studies help calculate these losses, allowing engineers to identify areas where losses can be reduced through techniques like reactive power compensation or network reconfiguration.
- **Improving economic operation:** By optimizing power flow and minimizing losses, load flow studies contribute to the economic operation of the power system, reducing generation costs and improving overall efficiency.

Why are Load Flow Studies Important?

- **Planning:** Load flow studies are essential for planning future expansions or modifications to the power system. They help engineers determine the best locations for new generation, transmission lines, and substations.
- **Operation:** During day-to-day operation, load flow studies help ensure the system is operating within safe limits. They can be used to optimize generator output, adjust transformer tap settings, and manage reactive power flow.
- **Security assessment:** Load flow studies are used to assess the impact of potential contingencies (e.g., the outage of a transmission line or generator) on the system's stability and reliability.
- **Equipment selection:** The results of load flow studies inform the selection of appropriate equipment ratings and characteristics for new installations.

load flow studies are indispensable for:

- **Safe operation:** Preventing voltage problems and equipment overloads.
- **Reliable supply:** Ensuring continuous power delivery even during contingencies.
- **Efficient operation:** Minimizing losses and optimizing resource utilization.
- **Future planning:** Guiding system expansion and development.

Without load flow studies, power systems would be vulnerable to a wide range of problems, potentially leading to instability, blackouts, and economic losses.

How Load Flow Studies Are Performed

1. **System Modeling:** A detailed mathematical model of the power system is created, including transmission lines, transformers, generators, loads, and their electrical characteristics.
2. **Data Collection:** Data on loads, generation, and system configuration are gathered.
3. **Solution Methods:** Numerical techniques (like the Gauss-Seidel or Newton-Raphson methods) are used to solve the power flow equations.
4. **Analysis:** The results are analyzed to identify potential problems, such as voltage violations, overloads, or excessive losses.

the typical steps involved in a power flow (load flow) study:

1. Data Collection and Preparation

- **System Data:** This is the foundation of the study. It includes:
  - **Network Topology:** A detailed description of how all the components (buses, transmission lines, transformers, generators, loads) are connected. This is often represented by a one-line diagram or single line diagram
  - **Electrical Parameters:** Impedances of transmission lines and transformers, generator characteristics (reactive power capabilities, voltage control settings), load characteristics (power factor, voltage dependence), etc.
  - **Load Data:** Real and reactive power demand at each load bus. This data can be based on historical records, forecasts, or real-time measurements.
  - **Generation Data:** Real and reactive power output of generators, along with their voltage control settings.
- **Software Selection:** Power flow studies are usually performed using specialized software tools (e.g., ETAP, PSS/E, PowerFactory). The software helps manage the complex calculations and provides a user-friendly interface for analysis.

2. System Modeling

- **Creating the Network Model:** The collected data is used to create a mathematical model of the power system within the chosen software. This involves representing each component (lines, transformers, generators, loads) with its appropriate electrical model.
- **Bus Classification:** Buses (connection points) in the system are classified into three types:
  - **PQ Buses (Load Buses):** Real and reactive power are specified. Voltage magnitude and angle are to be determined.
  - **PV Buses (Generator Buses):** Real power and voltage magnitude are specified. Reactive power and voltage angle are to be determined.
  - **Slack Bus (Swing Bus):** Voltage magnitude and angle are fixed. This bus serves as a reference point for the system and balances any mismatch between generation and load.

3. Formulation of Power Flow Equations

- **Nodal Equations:** The power flow problem is formulated as a set of non-linear algebraic equations based on Kirchhoff's laws and the power balance equations at each bus. These equations relate the bus voltages, power flows, and network parameters.

4. Solution of Power Flow Equations

- **Numerical Methods:** Because the power flow equations are non-linear, they are solved using iterative numerical techniques. Common methods include:
  - **Gauss-Seidel Method:** A relatively simple iterative method, but it can be slow to converge for large systems.

- Newton-Raphson Method: A more sophisticated method that converges faster than Gauss-Seidel, but requires more computational resources.
- **Fast Decoupled Method:** An approximation of the Newton-Raphson method that is even faster, particularly for large systems.

5. Analysis of Results

- **Voltage Profile:** The software calculates the voltage magnitude and angle at each bus. This is crucial for ensuring voltages are within acceptable limits.
- **Power Flows:** The real and reactive power flow through each transmission line and transformer is determined. This helps identify potential overloads.
- **Losses:** The real and reactive power losses in the system are calculated, allowing for efficiency analysis.
- **Equipment Loading:** The loading of each piece of equipment is assessed to ensure it's operating within its thermal limits.

6. Interpretation and Recommendations

- **Identify Problems:** Based on the results, engineers identify any potential problems, such as voltage violations, overloads, or excessive losses.
- **Develop Solutions:** If problems are identified, they develop solutions, such as adjusting generator output, changing transformer tap settings, or reconfiguring the network.
- **Documentation:** The results of the load flow study, along with any recommendations, are documented in a comprehensive report.

Important Notes:

- **Iterative Process:** The solution of power flow equations is iterative. The numerical methods start with initial guesses for the unknown variables and then refine these guesses until a converged solution is reached.
- **Convergence:** The convergence of the numerical method is important. A non-converging solution indicates that there might be problems with the system model or input data.
- **Software Tools:** Modern power flow studies rely heavily on specialized software tools that automate the process and provide advanced analysis capabilities.

Summary:

1. Representation of power system network by single line diagram

2. Obtain the impedance diagram

3. Obtaining the non linear algebraic power flow equations

4. Solving of non linear equation by using Iterative methods.

At every busbar (substation) or node containing four parameters P,Q,|V|,|δ|.out of thes four parameters,2 parameters are known and other 2 parameters are to be calculated using Numerical methods

- **Gauss-Seidel Method:** A relatively simple iterative method, but it can be slow to converge for large systems.
- **Newton-Raphson Method:** A more sophisticated method that converges faster than Gauss-Seidel, but requires more computational resources.
- **Fast Decoupled Method:** An approximation of the Newton-Raphson method that is even faster, particularly for large systems.

In **load flow (power flow) studies**, buses represent nodes in an electrical power system where power generation, consumption, or transmission occurs. There are three main types of buses:

1. PQ (Load) Bus

- Represents loads (power consumption points).
- Given: **Real power (P) and reactive power (Q) (demand values).**
- Unknown: **Voltage magnitude (|V|) and phase angle (θ).**
- Example: A substation supplying loads or a customer connection point.

2. PV (Generator) Bus or Voltage controlled bus

- Associated with power generation sources (like generators).
- Given: **Real power (P) and voltage magnitude (|V|).**
- Unknown: **Reactive power (Q) and phase angle (θ).**
- The generator adjusts reactive power to maintain voltage.
- Example: A bus with a generator regulating voltage.

3. Slack (Swing) Bus or Reference Bus

- One per system (usually).
- Maintains system voltage and supplies/extracts excess real and reactive power to balance the power flow equations.
- Given: **Voltage magnitude (|V|) and phase angle (θ).**
- Unknown: **Real power (P) and reactive power (Q).**
- Example: A large generator or substation.
- (one of the generator bus is made to take the additional real and reactive power to supply transmission losses) **Summary Table**

S.NO	Bus Type	Known Values	Unknown Values
1	Slack Bus	|V|,|δ|	P,Q
2	PV Bus	P,|V|	Q, |δ|
3	PQ Bus	P,Q	|V|,|δ|

Gauss Siedel Method

The load flow equations are Non-linear and they can be Solved by an iterative methods. one of the iterative metho is G S method. Compared to Gauss method it reduces no of iterations.

For load bus P, Q are known and V and δ are unknown values need to be calculated. The reference bus or Slack bus values are remains unchanged till the computational process reaches to its end.

let n is total no of buses

(n-1) load buses [$\because$ 1 is slack bus]

Complex power injected at i^{th} bus is given by $S_i = V_i I_i^*$

$$S_i^* = V_i^* I_i = P_i - jQ_i \quad -①$$

$$\hookrightarrow I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$\boxed{I = YV}$$

$$I_1 = [Y_{11} + Y_{12} + Y_{13} \cdots + Y_{1n}] Y_K$$

from eqn 1 $\quad I_i = \sum_{K=1}^{n} Y_{iK} V_K$

$$\Rightarrow I_i = \frac{P_i - jQ_i}{V_i^*} = Y_{i1}V_1 + Y_{i2}V_2 + Y_{i3}V_3 + \cdots + Y_{ii}V_i + \cdots + Y_{in}V_n$$

$$= Y_{11} V_i + \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K \Rightarrow I_i - \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K = Y_{11} V_i$$

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K \right]$$

r, r+1 iteration numbers.

$$\boxed{V_i^{r+1} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K \right]} \rightarrow \text{Gauss Siedel eqn}$$

For PQ buses.

Gauss Siedel Method

The load flow equations are non-linear and they can be solved by an iterative methods. One of the iterative metho is G.S method. Compared to Gauss method it reduces no of iterations.

For load bus P, Q are known and V and δ are unknown values need to be calculated. The reference bus or slack bus values are remains unchanged till the computational process reaches to its end.

let n is total no of buses

$(n-1)$ load buses [$\because$ 1 is slack bus]

Complex power injected at i^{th} bus is given by $S_i = V_i I_i^*$

$$S_i^* = V_i^* I_i = P_i - jQ_i \quad -(1)$$

$$\hookrightarrow I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$\boxed{I = YV.}$$

$$I_1 = [Y_{11} + Y_{12} + Y_{13} \cdots + Y_{1n}] Y_K$$

from eqn 1 $\quad I_i = \sum_{K=1}^{n} Y_{iK} V_K$

$$\Rightarrow I_i = \frac{P_i - jQ_i}{V_i^*} = Y_{i1}V_1 + Y_{i2}V_2 + Y_{i3}V_3 + \cdots + Y_{ii}V_i + \cdots + Y_{in}V_n.$$

$$= Y_{11} V_i + \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K. \Rightarrow I_i - \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K = Y_{11} V_i$$

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K \right]$$

$r, r+1$ iteration numbers.

$$\boxed{V_i^{r+1} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{\substack{K=1 \\ K \neq i}}^{n} Y_{iK} V_K \right]} \rightarrow \text{Gauss Siedel eqn}$$

For PQ buses.

Case 1): Networking containing 1 slack bus and all are PQ buses.

Algorithm:→

i). From the given n/w calculate Ybus matrix

ii). For the load buses calculate P_i, Q_i from the given values of P_g, P_D, Q_g & Q_D → $P_i = P_G - P_D$; $Q_i = Q_G - Q_D$

iii). Assume initial values of V and δ for the load buses.

iv). Calculate V and δ by using G.S eqns.

v). Calculate the magnitude difference voltages b/w two successive iterations. if this difference is zero stop the iteration otherwise continue the iteration until the difference is become zero. Apply the same procedure for the all load buses and calculate V and δ

vi). by applying static power flow equations calculate P and Q for the slack bus.

$$P_i = \sum_{K=1}^{n} |Y_{iK} V_K V_i| \cos(\theta_{iK} + \delta_K - \delta_i)$$

$$Q_i = -\sum_{K=1}^{n} |Y_{iK} V_K V_i| \sin(\theta_{iK} + \delta_K - \delta_i).$$

P_i and Q_i are steady state power flow equations

Case 2: Network Containing 1 slack bus 1 PV bus and all the remaining are of load buses. and the calculated value of the Q for the PV buses is within the given range of Q for the PV bus.

Algorithm:

i) Calculate Ybus for the given n/w.

ii) Assume the initial values V's for PQ bus and δ for the PV bus.

iii) Calculate Q for PV bus using static power flow eqns

$$-Q_i = \sum_{K=1}^{n} |Y_{iK} V_K V_i| \sin(\theta_{iK} + \delta_K - \delta_i)$$

iv). Check whether the calculated of Q is within the range then the PV bus. If it is with in the given range [Q_{max} to Q_{min}] then the PV bus works as PV bus and maintaining constant value of voltage.

v). Calculate δ for the PV bus using G.S eqn

$$\boxed{V_i^{\,r+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{\,r})^*} - \sum_{K=1}^{n} Y_{iK} V_K\right]} \Rightarrow \text{G.S eq}^n$$

vi). Calculate V and δ for the load bus using G.S eqn

vii). Calculate P and Q for the slack bus using static power flow eqns.

$$\boxed{\begin{aligned} P_i &= \sum_{K=1}^{n} |Y_{iK} V_K V_i| \cos(\theta_{iK} + \delta_K - \delta_i) \\ -Q_i &= \sum_{K=1}^{n} |Y_{iK} V_K V_i| \sin(\theta_{iK} + \delta_K - \delta_i) \end{aligned}}$$

Case 3: Network containing 1 slack bus 1 PV bus and Q-value for the PV bus is in the out of range for a given n/w value of Q so that PV bus is working as a PQ bus. with the value of Q for this PQ bus is the nearest value from the calculated value of Q in the given range.

Algorithm:→

i). Repeat the step 1,2,3 similar to the case 2.

ii). Check whether the calculate value is in the given range of Q, if it is not in the given range then consider PV bus as PQ bus.

iii) Repeat the steps similar to the case i

* Accelerating factor (α):→

It is used for reducing the no of iterations using G.S method

$$\boxed{V_{i\,(acc)}^{(q+1)} = V_i^{(q)} + \alpha\left[V_i^{(q+1)} - V_i^{(q)}\right]}$$

→ To speed up the operation and it is a real number. For a suitable vale of α for trial load flows is 1.6.

→ The System data for a load flow solutions are given below table. (3)

The line admittances:

Bus Code	Admittance
1−2	2−j8
1−3	1−j4
2−3	0.666 − j2.664
2−4	1−j4
3−4	2−j8

The Schedule of active and reactive powers

Bus Code	P	Q	V	Remarks
1	—	—	1.06	slack
2	0.5	0.2	1+j0.0	PQ
3	0.4	0.3	1+j0.0	PQ
4	0.3	0.1	1+j0.0	PQ

Determine the voltages at the end of first iteration using G.S. method. Take $\alpha = 1.6$

Sol :→ i). Determine the admittance matrix. no of buses = 4

$$Y_{bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} & -Y_{14} \\ -Y_{21} & Y_{22} & -Y_{23} & -Y_{24} \\ -Y_{31} & -Y_{32} & Y_{33} & -Y_{34} \\ -Y_{41} & -Y_{42} & -Y_{43} & Y_{44} \end{bmatrix}_{4\times4}$$

$$Y_{11} = Y_{12} + Y_{13} = 2-j8 + 1-j4 = 3-j12$$

$$Y_{22} = Y_{21} + Y_{23} + Y_{24} = 2-j8 + 0.666 - j2.664 + 1-j4$$

$$Y_{22} = 3.666 - j14.664$$

$$Y_{33} = Y_{31} + Y_{32} + Y_{34} = 1-j4 + 0.666 - j2.664 + 2-j8$$

$$Y_{33} = 3.666 - j14.664.$$

$$Y_{44} = Y_{42} + Y_{43} = 1-j4 + 2-j8 = 3-j12.$$

Mutual admittances

$Y_{12} = Y_{21} \Rightarrow -Y_{12} = -(2-j8) = -2+j8$.

$Y_{13} = Y_{31} \Rightarrow -Y_{13} = -(1-j4) = -1+j4$

$Y_{14} = Y_{41} \Rightarrow -Y_{14} = 0$.

$Y_{23} = Y_{32} \Rightarrow -Y_{23} = -(0.666 - j2.664) = -0.666 + j2.664$

$Y_{24} = Y_{42} \Rightarrow -Y_{24} = -(1-j4) = -1+j4$

$Y_{34} = Y_{43} \Rightarrow -Y_{34} = -(2-j8) = -2+j8$.

$$Y_{bus} = \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix}$$

ii). → Calculate P_i and Q_i

Now At bus 2 $P_2 = P_{G2} - P_{D2} = 0 - 0.5 = -0.5$ p.u

$Q_2 = Q_{G2} - Q_{D2} = 0 - 0.2 = -0.2$ pu

At bus 3, $P_3 = P_{G3} - P_{D3} = 0 - 0.4 = -0.4$ p.u

$Q_3 = Q_{G3} - Q_{D3} = 0 - 0.3 = -0.3$ pu

At bus 4, $P_4 = P_{G4} - P_{D4} = 0 - 0.3 = -0.3$ p.u

$Q_4 = Q_{G4} - Q_{D4} = 0 - 0.1 = -0.1$ p.u

→ iii). Assume initial values of V and δ for the load buses

Bus - 1 slack bus $V_1 = 1.06$

$V_2^0 = V_3^0 = V_4^0 = 1\angle 0^\circ$

For first iteration $r = 0$

→ iv). Calculate V and δ by using G.S eqn

$$V_i^{q+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^q)^*} - \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik} V_k\right]$$

Here bus $(i) = 2$; and $n = 4$.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.5 - j(-0.2)}{1-j} - (2+j8)(1+0.6) - (-0.666 + j2.664)\,1 - (-1+j4)\,1\right]$$

$$= \frac{1}{3.666 - j14.664}\left[-0.5 + j0.2 + 2.12 - j8.48 + 0.666 - j2.664 + 1 - j4\right]$$

$$= \frac{1}{3.666 - j14.664}\left[3.286 - j14.944\right]$$

$$= (1.01187 - j0.02888) \text{ p.u.}$$

→ Calculate the magnitude difference voltage b/w two successive iterations

$$\Delta V_2^1 = V_2^1 - V_2^0 = 1.01187 - j0.02888 - (1 + j0) = 0.01187 - j0.02888 \text{ p.u.}$$

Accelerated factor voltage

$$V_{2\,acc}^1 = V_2^0 + \alpha\left[V_2^1 - V_2^0\right] \quad (\Delta V_2^1)$$

$$= 1 + j0 + 1.6\,(0.01187 - j0.02888)$$

$$= 1 + 0.018992 - j0.046208 \text{ p.u.}$$

$$= 1.018992 - j0.046208 \text{ p.u.}$$

The voltage at bus 3 is

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= 0.994119 - j0.029248.$$

$$V_{3_{acc}}^1 = V_3^0 + \alpha(V_3^1 - V_3^0)$$

$$= 1 + j0 + 1.6(0.994119 - j0.029248 - 1 - j0.0)$$

$$= 0.999059 - j0.0467968 \text{ p.u.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - \cancel{Y_{41}V_1^0} - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= 0.9716032 - j0.064684$$

$$V_4^0 + \alpha(V_4^1 - V_4^0)$$

$$V_{4acc}^1 = 1.0 + j0.0 + 1.6[0.9716032 - j0.064684 - 1 - j0]$$

$$V_{4acc}^1 = 0.954565 - j0.1034944 \text{ p.u.}$$

→ For the above problem bus 2 is taken as a generator bus with $|V_2| = 1.04$ and reactive power constraint is $0.1 \leq Q_2 \leq 1.0$

Determine the voltages starting with a flat voltage profile and assuming accelerating factor as 1.0.

Sol:→ From the given data bus 2 is taken as a generator bus (PV)

Q_2 is not specified and $P_2 = 0.5$.

5

To find V_2' we first find Q_2 with $V_2 = 1.04 + j0.0$ & phase angle '0'.

$$P_2 - jQ_2 = V_2^* \sum_{q=1}^{4} Y_{2q} V_q \qquad \simeq P_i - jQ_i = \sum_{K=1}^{n} Y_{iq} V_q V_i^*$$

$$Q_2 = -\text{Img}\left[V_2^{0*} \left(Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 \right] \right.$$

$$= -\text{Img}\left[(1.04 - j0.0)(-2 + j8.0)(1.06) + 3.666 - j14.664(1.04) \right.$$
$$\left. + (-0.666 + j2.664)(1 + j0.0) + (-1 + j4.0)\,1.0 \right] = 0.1108$$

Since Q_2 lies within the limits

$\therefore \quad V_2 = |V_2|$ spec.

$$V_2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0 \right]$$

Bus 2 is generator bus so P_2 and Q_2 are to be +ve.

so $Q_2 = +0.1108$.

$$V_2 = \frac{1}{3.666 - j14.664} \left[\frac{0.5 - j0.1108}{1.04 - j0.0} - (-2 + j8.0)(1.06) - (-0.666 + j2.664)1.0 - (-1 + j4.0)\,1.0 \right]$$

$V_2' = 1.0472846 + j0.0291476$;

$V_2' = 1.04 \angle 1.59^\circ$.

$$V_3' = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^*} - Y_{31}V_1 - Y_{32}V_2' - Y_{34}V_4^0 \right]$$

$$= \frac{1}{3.666 - j14.664} \left[\frac{-0.4 + j0.3}{1 - j0.0} - (-1 + j4)1.06 - (-0.666 + 2.664)(1.0395985 + j0.0289) - (-2 + j8)(1 + j0.0) \right]$$

$= 0.9978866 - j0.015607057$

Illy V_4' obtained $V_4' = 0.998065 - j022336$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{3-j12}\left[\frac{-0.3+j0.1}{1+j0.0} - 0.0\times 1.06 - (-1+j4)(1.0395985+j0.02891) - (-2+j8)(0.99788660 - j0.0156070 7)\right]$$

$V_4^1 = 0.998065 - j0.022336$.

→ * For the above problem (Case 1) if the reactive power constraint on generator 2 is $0.2 \le Q_2 \le 1.0$. Determine the voltages at the end of first iteration.

Sol :→ Since Q_2 calculated corresponding $V_2 = 1.04 + j0.0$ is 0.1108. which is lessthan the minimum specified $Q_{2min} = 0.2$ for the given problem.

→ The reactive power generator for bus 2 is fixed at 0.2.
and the bus is considered as load bus for this iteration.
voltage $V_2^0 = 1+j0.0$ For load bus P and Q are unknown and considered as $-ve$ values.
V and δ are $+ve$.

$$\therefore V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{0.5 - j0.2}{1 - j0.0} - (-2+j8.0)\,1.06 - (-0.666 + j0.2664) - (-1+j4.0)\,1.0\right]$$

$V_2^1 = 1.098221 + j0.0301056$

Illy V_3^1 and V_4^1 can be evaluated.
The no of iterations can be minimised by acceleration factor $\alpha = 1.6$

$$V_{i\,acc}^{r+1} = V_i^r + \alpha\left(V_i^{r+1} - V_i^r\right)$$

$i=2, r=0,$ $V_{2acc}^{1} = V_2^0 + 1.6\left[V_2^1 - V_2^0\right]$

$$= (1+j0.0) + 1.6\left[(1.0982+j0.0301) - (1+j0)\right]$$

$$V_{2acc}^{1} = (1.157 + j0.048)\ \text{p.u.}$$

Voltage at bus-3 at the end of 1^{st} iteration $i=3, r=0$.

$$V_i^{r+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{\substack{K=1 \\ K\neq i}}^{n} Y_{iK} V_K\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 + V_{32}V_{2ac}^{1} + Y_{34}V_4^0)\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4+j0.3}{(1+j0)^*} - (-1+j4)(1.06) + (-0.666+j2.664)\right.$$
$$\left.(1.157+j0.048)(1+j0)(-2+j8)\right]$$

$$= \frac{1}{3.666 - j14.664}\left[-0.4+j0.3 - (-1.06+j0.24 - 2+j8 - 0.7705 - \right.$$
$$\left. 0.1278 + j3.0822 - j0.03196)\right]$$

$$V_3^1 = (1.019 - j0.028)\ \text{p.u.}$$

$i=3, r=0$ $\quad V_{3acc}^{r+1} = V_i^r + \alpha\left[V_i^{r+1} - V_i^r\right]$

$$= (1+j0) + 1.6\left[(1.019 - j0.028) - (1+j0)\right]$$

$$V_{3acc}^{1} = 1.0304 - j0.0448\ \text{p.u.}$$

Voltage at bus 4 at the end of 1^{st} iteration $i=4, r=0$.

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 + V_{42}V_{2acc}^{1} + Y_{43}V_{3acc}^{1}\right]$$

$$= \frac{1}{3-j12}\left[\frac{-0.3-(-j0.1)}{1+j0.0} - (0.1.06 + (-1+j4)(1.157+j0.048)\right.$$
$$\left. + (-2+j8)(1.0304 - j0.0448)\right]$$

$$= \frac{1}{3-j12}\left[-0.3+j0.1 - (-1.349+j4.58 - 1.7024 + j8.3328)\right]$$

$$= \frac{1}{3-j12}\left(27514 - j12.8128\right) = \frac{13.104\angle -77.87}{12.369\angle -75.96}$$

$$V_4^1 = 1.058 - j0.035 \text{ pu.}$$

The no of iterations can be minimised by acceleration factor

$$V_{i\,acc}^{r+1} = V_i^r + \alpha\left[V_i^{r+1} - V_i^r\right]$$

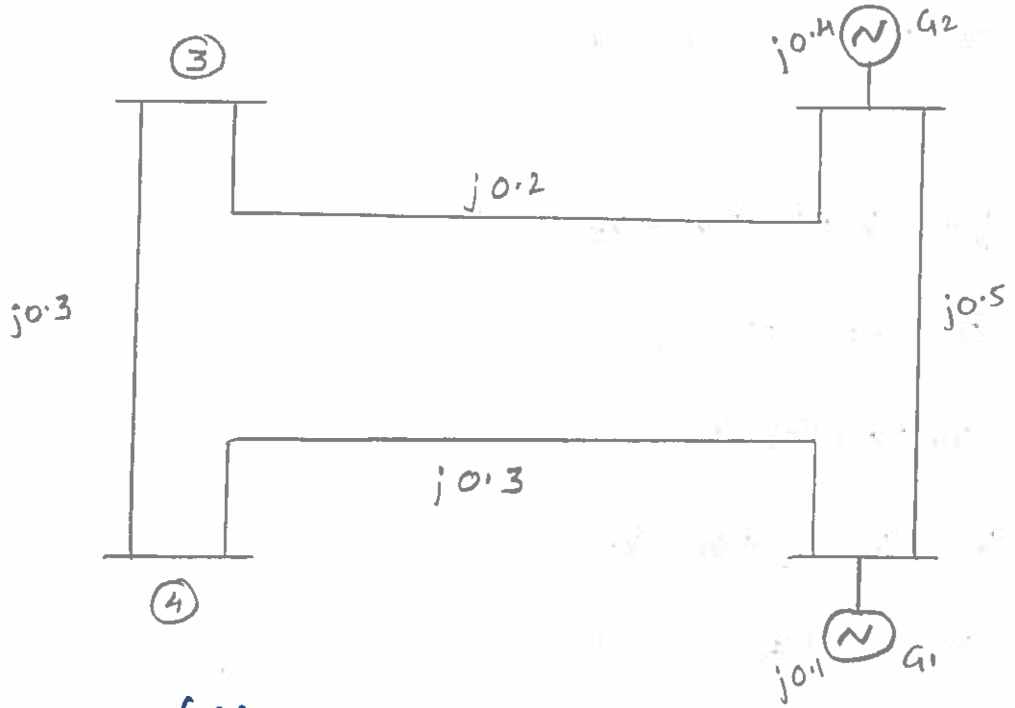
$i=4,\ r=0$

$$V_{4\,acc}^{1} = V_4^0 + 1.6\left[V_4^1 - V_4^0\right]$$

$$= (1+j0) + 1.6\left[(1.058 - j0.035) - (1+j0)\right]$$

$$V_{4\,acc}^{1} = (1.0928 - j0.056) \text{ p.u.}$$

1. Form the Y-BUS for the network shown in the Fig Including Generated Buses 1 and 2 with Impedance of 0.1, 0.4 per unit respectively all the values are per unit Impedances for the network.



$$Y_{BUS} = \begin{pmatrix} Y_{11} & -Y_{12} & -Y_{13} & -Y_{14} \\ -Y_{21} & Y_{22} & -Y_{23} & -Y_{24} \\ -Y_{31} & -Y_{32} & Y_{33} & -Y_{34} \\ -Y_{41} & -Y_{42} & -Y_{43} & Y_{44} \end{pmatrix}$$

$$y_1 = \frac{1}{z_1} = \frac{1}{j0.1} = -j10 \text{ pu}$$

$$y_2 = \frac{1}{z_2} = \frac{1}{j0.4} = -j2.5 \text{ pu}$$

$y_{12} = y_{21} = \frac{1}{j0.5} = -j2\,PU$

$y_{13} = y_{31} = 0$

$y_{14} = y_{41} = \frac{1}{j0.3} = -j3.33\,PU$

$y_{32} = y_{23} = \frac{1}{j0.2} = -j5\,PU$

$y_{24} = y_{42} = 0$

$y_{34} = y_{43} = \frac{1}{j0.3} = -j3.33\,PU$

NOW:

$\boxed{y_{11} = y_1 + y_{12} + y_{13} + y_{14}}$

$y_{11} = -j10 - j2 - 0 - j3.33$

$y_{11} = -j15.33\,PU$

$\boxed{y_{22} = y_2 + y_{21} + y_{23} + y_{24}}$

$y_{22} = -j2.5 - j2 - j5 + 0$

$y_{22} = -j9.5\,PU$

$\boxed{y_{33} = y_{31} + y_{32} + y_{34}}$

$= 0 - j5 - j3.33$

$y_{33} = -j8.33\,PU$

$\boxed{y_{44} = y_{41} + y_{42} + y_{43}}$

$= -j3.33 - 0 - j3.33$

$y_{44} = -j6.66\,PU$

$$\therefore\ Y_{BUS} = \begin{bmatrix} -j15.33\,pu & j2\,pu & 0 & j3.33\,pu \\ j2\,pu & -j9.5\,pu & j5\,pu & 0 \\ 0 & j5\,pu & j8.33\,pu & j3.33\,pu \\ j3.33\,pu & 0 & j3.33\,pu & j6.66\,pu \end{bmatrix}$$

$$\boxed{\text{Sparsity } (x\%) = \frac{\text{No. of zero elements}}{\text{Total no. of elements}} \times 100}$$

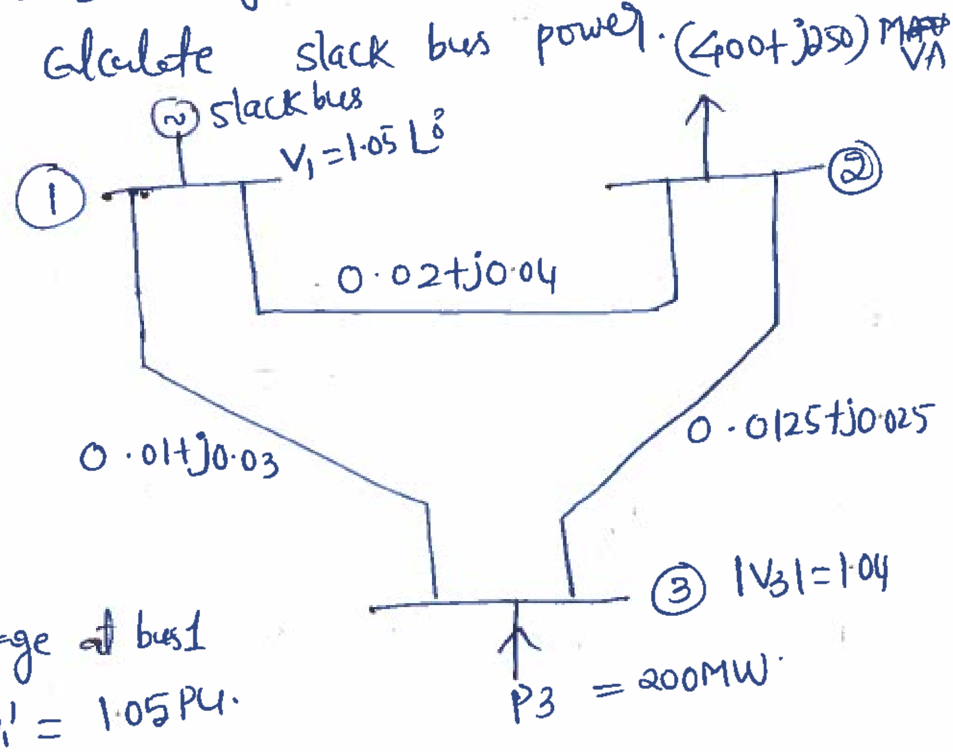
$$\therefore\ \text{sparsity} = \frac{4}{16} \times 100 = 25\%$$

$$\boxed{\text{Transmission lines} = \frac{n^2(1-x)-n}{2}}$$

$$= \frac{16(1-0.25)-4}{2}$$

$$\therefore\ \text{Transmission lines} = 4$$

→ * Single line diagram of a simple power system, with generators at buses 1 and 3 is shown in figure. The magnitude of voltage at bus 1 is 1.05 p.u. Voltage magnitude at bus 3 is fixed at 1.04 p.u with active power generation of 200MW. A load consisting of 400 MW and 250MVAR is taken from bus 2. line impedances are marked in p.u on a 100 MVA base and the line charging susceptances are neglected. Determine the voltage at bus 2 and 3 using G.S method at the end of first iteration Also calculate slack bus power.



Sol :→

Magnitude of voltage at bus 1

is $V_1^0 = V_1^1 = 1.05$ PU.

$|V_3| = 1.04$.

Base MVA = 100 MVA.

Assume a bus 2 flat voltage for PQ bus is $V_2^0 = 1 + j0.0$

Step 1 :→ Form Ybus matrix

$$n=3 \quad Y_{bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} \\ -Y_{21} & Y_{22} & -Y_{23} \\ -Y_{31} & -Y_{32} & Y_{33} \end{bmatrix}_{3\times3}$$

$$Y_{11} = Y_{12} + Y_{13} = \frac{1}{Z_{12}} + \frac{1}{Z_{13}} = \frac{1}{0.02+j0.04} + \frac{1}{0.01+j0.03}$$

$$= 20 - j50$$

$$Y_{22} = Y_{21} + Y_{23} = \frac{1}{0.02+j0.04} + \frac{1}{0.0125+j0.025} = 26 - j52$$

$$Y_{33} = Y_{31} + Y_{32} = \frac{1}{Z_{31}} + \frac{1}{Z_{32}}$$

$$= \frac{1}{0.01+j0.03} + \frac{1}{0.0125+j0.025} = 26-j62.$$

$$Y_{12} = Y_{21} \Rightarrow -Y_{12} = \frac{-1}{Z_{12}} = \frac{-1}{0.02+j0.04} = -10+j20$$

$$Y_{13} = Y_{31} \Rightarrow -Y_{31} = -Y_{13} \Rightarrow -10+j30$$

$$Y_{32} = Y_{23} \Rightarrow -Y_{32} = -Y_{23} = -16+j32.$$

$$Y_{bus} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j32 \end{bmatrix}$$

Now at bus 2 (PQ) $P_2 = P_{G2} - P_{D2} = 0 - \frac{400}{100} = -4\,pu.$

$$Q_2 = Q_{G2} - Q_{D2} = 0 - \frac{250}{100} = -2.5\,pu.$$

PV bus :→ $P_3 = P_{G3} - P_{D3} = \frac{200}{100} - 0 = 2\,pu.$

W.K.T $$V_i^{r+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{\substack{K=1 \\ K\neq i}}^{n} Y_{ik} V_k\right]$$

For first iteration $r=0$; $i=2$.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^0 - Y_{23}V_3^0\right] \qquad V_1^0 = 1.05, \quad V_3^0 = 1.04$$

$$V_2^1 = 0.9746 - j\,0.0423$$

bus 3 is PV bus & voltage controlled bus. So, reactive power is calculated as

$$Q_3^1 = -\left[V_i^* Y_{iq} V_q \sin(\theta_{ik} + \theta_K - \theta_i)\right]$$

$$= -\mathrm{Img}\left[(V_3^0)^* \left[Y_{31}V_1' + Y_{32}V_2' + Y_{33}V_3^0\right]\right]$$

$$= -\mathrm{Img}\left[(1.04)\left[(-10+j30)\,1.05 + (-16+j32)(0.9746-j0.0423) - (26-j62)(1.04)\right]\right]$$

$$= -\mathrm{Img}\left[2.392 - j1.16\right]$$

$$Q_3' = 1.16 \text{ pu.}$$

The voltage at bus 3 is

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3'}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2'\right]$$

$$= 1.03782 - j0.005165$$

$$V_3' = 1.03783 \angle -0.0285^\circ$$

$$\delta_3' = -0.285^\circ$$

Now $V_3' = |V_3|_{\text{specified}} \angle \delta_3' = 1.04 \angle -0.285$

$$= 1.039987 - j0.00517$$

Slack bus power :-

$$S_i^* = P_i - jQ_i = V_i^* \sum_{q=1}^{n} Y_{pq} V_q.$$

$i = 1, \; n = 3.$

$$P_1 - jQ_1 = V_1^*\left[Y_{11}V_1 + Y_{12}V_2' + Y_{13}V_3'\right]$$

$V_2' = 0.9746 - j0.0423$

$V_3' = 1.039987 - j0.0517$

$$P_1 - jQ_1 = 1.9488 - j1.484.$$

Real power(P) $= 1.9488 \times 100 = 194.88$ MW.

Reactive power(Q) $= 1.4 \times 100 = 140$ MVAR.

→ The load flow data for the sample power system are given below. The voltage magnitude at bus 2 is to be maintained at 1.04 p.u. The maximum and minimum reactive power limits of the generator at bus 2 are 0.35 and 0.0 p.u respectively. Determine the set of load flow equations at the end of first iteration by N.R method.

Impedance for Sample System:

Bus Code	Impedance	line charging admittance
1-2	0.08+j0.24	0.0
1-3	0.02+j0.06	0.0
2-3	0.06+j0.18	0.0

Schedule of generation & loads:

Bus code	Assumed voltages	Generation MW	Generation MVAR	Load MW	Load MVAR
1	1.06+j0.0	0	0	0	0
2	1.0+j0.0	0.2	0	0	0
3	1.0+j0.0	0	0	0.6	0.25

Sol :→

$$Y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.08+j0.24} \times \frac{0.08-j0.24}{0.08-j0.24}$$

$$Y_{12} = 1.25 - j3.75$$

Illy $Y_{13} = 5 - j15$ and $Y_{23} = 1.667 - j5.0$

$$Y_{bus} = \begin{bmatrix} 6.25-j18.75 & -1.25+j3.75 & -5+j15 \\ -1.25+j3.75 & 2.916-j8.75 & -1.666+j5.0 \\ -5+j15 & -1.666+j5.0 & 6.666-j20 \end{bmatrix}$$

Assuming a flat voltage profile for bus 2 and 3 and for bus 1

$$V_1 = 1.06 + j0.0$$

For the nodal admittance matrix and assuming voltage solution $\boxed{Y = G - jB}$

$G_{11} = 6.25$; $B_{11} = 18.75$

$G_{12} = -1.25$ $B_{12} = -3.75$ $\qquad e_1 = 1.06 \qquad f_1 = 0.0$

$G_{13} = -5.0$; $B_{13} = -15.0$ $\qquad e_2 = 1.0 \qquad f_2 = 0.0$

$G_{22} = 2.916$; $B_{22} = 8.75$ $\qquad e_3 = 1.0 \qquad f_3 = 0.0$

$G_{23} = -1.666$; $B_{23} = -5.0$

$G_{33} = 6.666$; $B_{33} = 20.$

$$P_p = \sum_{q=1}^{n} \{ e_p (e_q G_{pq} + f_q B_{pq}] + f_p (f_q G_{pq} - e_q B_{pq}) \}$$

$p = 2$; $q = 1, 2, 3$

$$P_2 = \{ e_2 (e_1 G_{21} + f_1 21) + f_2 (f_1 G_{21} - e_1 B_{21})$$
$$+ e_2 (e_2 G_{22} + f_2 B_{22}) + f_2 (f_2 G_{22} - e_2 B_{22})$$
$$+ e_2 (e_2 G_{23} + f_2 G_{23}) + f_2 (f_3 G_{23} - e_3 B_{23}) \}$$

$$P_2 = -0.075 \text{ p.u.}$$

lly $P_3 = -0.3$

$$Q_p = \sum_{q=1}^{n} \{ f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq}) \}$$

$$Q_2 = \{ \cancel{f_2}_0 (\quad) - e_2 (f_1 G_{21} - e_1 B_{21}) + f_2 (0)$$
$$- e_2 (f_2 G_{22} - e_2 B_{22}) + f_2 (0)$$
$$- e_2 (f_3 G_{23} - e_3 B_{23})$$

$$Q_2 = -0.225 \text{ p.u}$$

lly $Q_3 = -0.9$

$$\Delta P_2 = P_{2\,sp} - P_{2\,cal}$$
$$= 0.2 - (-0.075) = 0.275 \text{ pu}$$

$$\Delta P_3 = -0.6 - (-0.3) = -0.3 \text{ p.u}$$

Since the lower limit on Q_2 is 0.0 and the value of Q_2 as Calculate above Violates this limit, So bus 2 is treated as load bus $Q_{2sp} = 0.0$

$$\Delta Q_2 = \overset{Q_{spec}}{0.0} - \overset{Q_{cal.}}{(-0.225)} = 0.225.$$

$$\Delta Q_3 = -0.25 - (-0.9) = 0.65.$$

Diagonal elements ⇒

$$\frac{\partial P_p}{\partial e_p} = 2 e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial P_2}{\partial e_2} = 2 e_2 G_{22} + e_1 G_{21} + f_1 B_{21} + e_3 G_{23} + f_3 B_{23}$$

$$= 2 \times 1.0 \times 2.916 + 1.06(-1.25) + 0.0(-3.75) + 1.0(-1.666) + 0.0(-5.0) = 2.848.$$

$$\frac{\partial P_3}{\partial e_3} = 2 e_3 G_{33} + e_1 G_{31} + f_1 B_{31} + e_2 G_{32} + f_2 B_{32}$$

$$\frac{\partial P_3}{\partial e_3} = 6.3666.$$

$$\rightarrow \frac{\partial P_p}{\partial f_p} = \overset{2 f_p G_{pp}}{2 f_p G_{pp}} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\frac{\partial P_2}{\partial f_2} = 2 f_2 G_{22} + f_1 G_{21} - e_1 B_{21} + f_3 G_{23} - e_3 B_{23}.$$

$$\Rightarrow \cancel{\quad} = 8.975$$

$$\boxed{\frac{\partial P_p}{\partial f_p} = 2 f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})}$$

$$\frac{\partial P_3}{\partial f_3} = 20.90.$$

off diagonal elements :→

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}$$

$$\frac{\partial P_2}{\partial e_3} = e_2 G_{23} - f_2 B_{23} = -1.666.$$

$$\frac{\partial P_3}{\partial e_q} = -1.666, \quad \frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq}$$

$$\partial P_2 / \partial f_3 = -5.0; \quad \frac{\partial P_3}{\partial f_2} = -5.0$$

Ily we find out the partial derivatives of the reactive Diagonal element.

$$\frac{\partial Q_p}{\partial e_p} = 2e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\frac{\partial Q_2}{\partial e_2} = 8.525 \; ; \quad \frac{\partial Q_3}{\partial e_3} = 15.1$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq}).$$

$$\frac{\partial Q_2}{\partial f_2} = -2.991 \; ; \quad \frac{\partial Q_3}{\partial f_3} = -6.966$$

Ily $\frac{\partial Q_2}{\partial f_3}$; $\frac{\partial Q_3}{\partial f_2}$

The set of linear equations 1 are

$$\begin{bmatrix} 0.275 \\ -0.3 \\ 0.025 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 2.846 & -1.666 & 8.975 & -5.0 \\ -1.666 & 6.366 & -5.0 & 20.90 \\ 8.525 & -5 & -2.991 & 1.666 \\ -5.0 & 15.1 & 1.666 & -6.966 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} \end{bmatrix}$$

Ily Q

$$= \frac{1}{3-j12}\left(27514 - j12.8128\right) = \frac{13.104\angle -77.87}{12.369\angle -75.96}$$

$$V_4^1 = 1.058 - j0.035 \text{ p.u.}$$

The no of iterations can be minimised by acceleration factor

$$V_{i\,acc}^{r+1} = V_i^r + \alpha\left[V_i^{r+1} - V_i^r\right]$$

$i=4,\ r=0$

$$V_{4\,acc}^1 = V_4^0 + 1.6\left[V_4^1 - V_4^0\right]$$

$$= (1+j0) + 1.6\left[(1.058 - j0.035) - (1+j0)\right]$$

$$V_{4\,acc}^1 = (1.0928 - j0.056) \text{ p.u.}$$

the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \quad (24)$$

where α is a real number. When α =1, the value of $V_i^{(k-1)}$ is the computed value. If $1 < \alpha < 2$, then the value computed is extrapolated. Generally α is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

<u>Examples on GS load flow analysis:</u>

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if $V_1 = 1\angle 0^0$ pu.*

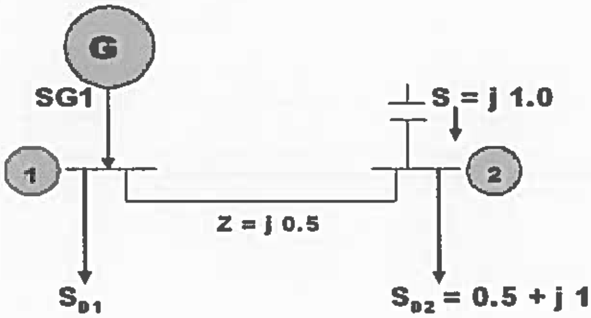


Fig : System of Example 1

<u>Solution</u>:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$S_2 = j1.0 - (0.5 + j\,1.0) = -0.5$ pu.

$V_1 = 1\angle 0^0$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - (j2 \times 1\angle 0^0)\right]$$

$$= 1.0 - j0.25 = 1.030776 \ \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - (j2 \times 1\angle 0^0)\right]$$

$$= 0.94118 - j\,0.23529 = 0.970145 \ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - (j2 \times 1\angle 0^0)\right]$$

$$= 0.9375 - j\,0.249999 = 0.970261 \ \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - (j2 \times 1\angle 0^0)\right]$$

$$= 0.933612 - j\,0.248963 = 0.966237 \ \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - (j2 \times 1\angle 0^0)\right]$$

$$= 0.933335 - j\,0.25 = 0.966237 \ \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10^{-6} pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472 \ \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472 \ \angle \ 14.931^0$$

$$= 0.5 + j\,0.133329 \text{ pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

$$= 0.517472 \ \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -0.5 + j\,0.0 \text{ pu}$$

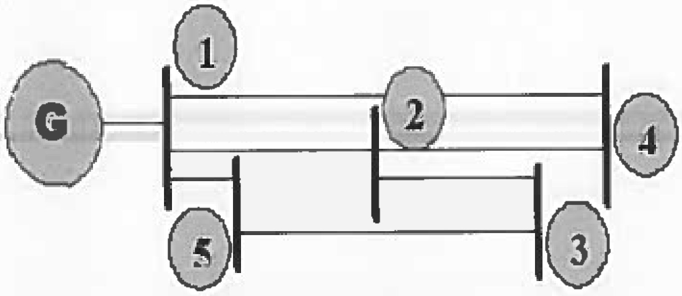
The total loss in the line is given by

$$S_{12} + S_{21} = j\,0.133329 \text{ pu}$$

Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{SP} \rvert$ (pu)	δ
1	-	-	-	-	1.02	0°
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

$P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $\quad P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

Y_{bus} =

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be V_1^0 = 1.02+j0.0 in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}\,V_1^o - Y_{23}\,V_3^0 - Y_{24}\,V_4^0 - Y_{25}\,V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294) \times 1.0\angle 0^o\}\right]$$

$$= 0.98140\angle -3.0665^o \quad = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3|\,|V_1|\,(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3|\,|V_2|\,(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2\,(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3|\,|V_4|\,(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3|\,|V_5|\,(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^o$; $\delta_2 = -3.0665^o$; $\delta_3 = 0^o$; $\delta_4 = 0^o$ and $\delta_5 = 0^o$

$\therefore\ \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^o$ ($\delta_{ik} = \delta_i - \delta_k$); $\quad \delta_{32} = 3.0665^o$

$Q_3 = 1.04\,[1.02\,(0.0+j0.0) + 0.9814\,\{-1.17647 \times \sin(3.0665^o) - 4.70588 \times\cos(3.0665^o)\} + 1.04\{-9.41176 \times\cos(0^o)\} + 1.0\,\{0.0 + j0.0\} + 1.0\{-4.70588\times\cos(0^0)\}]$

$= 1.04\,[-4.6735 + 9.78823 - 4.70588] \; = 0.425204$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}\,V_1^o - Y_{32}\,V_2^1 - Y_{34}\,V_4^0 - Y_{35}\,V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588) \times (0.98140\angle -3.0665^o)\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times (1\angle 0^o)\}\right]$$

$$= 1.05569\angle 3.077^o = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^0$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-0.58823 + j2.35294) \times (0.98140\angle -3.0665^o)\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle -7.303^o \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^o - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times 1.04\angle 3.077^o\}\right]$$

$$= 0.994618\angle -1.56^0 = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^0$ pu $\qquad V_2 = 0.98140\ \angle -3.066^0$ pu

$V_3 = 1.04\angle 3.077^0$ pu $\qquad V_4 = 0.955715\angle -7.303^0$ pu

and $\qquad V_5 = 0.994618\angle -1.56^0$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) *All buses except bus 1 are PQ Buses*

(ii) *Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu*

(iii) *Bus 2 is PV bus, with voltage magnitude specified as 1.04 and* $0.25 \le Q_2 \le 1.0$ *pu.*

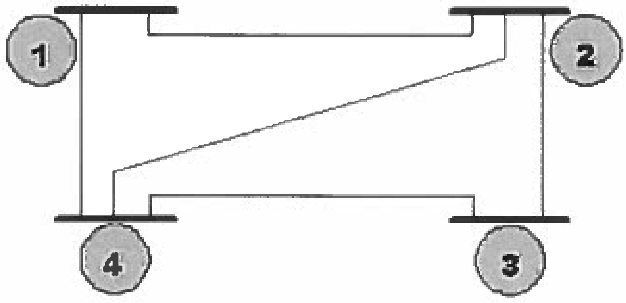


Fig. System for Example 3

<u>Table: Line data of example 3</u>

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

<u>Table: Bus data of example 3</u>

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04 \angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

<u>Solution</u>: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$$Y_{BUS} = \begin{bmatrix} 3.0 - j9.0 & -2.0 + j6.0 & -1.0 + j3.0 & 0 \\ -2.0 + j6.0 & 3.666 - j11.0 & -0.666 + j2.0 & -1.0 + j3.0 \\ -1.0 + j3.0 & -0.666 + j2.0 & 3.666 - j11.0 & -2.0 + j6.0 \\ 0 & -1.0 + j3.0 & -2.0 + j6.0 & 3.0 - j9.0 \end{bmatrix}$$

<u>Case(i)</u>: All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0 \angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}\,V_1^o - Y_{23}\,V_3^0 - Y_{24}\,V_4^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5+j0.2}{1.0-j0.0} - \{(-2.0+j6.0)\times(1.04\angle 0^o)\}\right.$$

$$\left. - \{(-0.666+j2.0)\times(1.0\angle 0^o)\} - \{(-1.0+j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.02014 \angle 2.605^0$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}\, V_1^o - Y_{32}\, V_2^1 - Y_{34}\, V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0-j0.5}{1.0-j0.0} - \{(-1.0+j3.0)\times(1.04\angle 0.0^o)\}\right.$$

$$\left. - \{(-0.666+j2.0)\times(1.02014\angle 2.605^o)\} - \{(-2.0+j6.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.03108 \angle -4.831^0$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}\, V_1^o - Y_{42}\, V_2^1 - Y_{43}\, V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3+j0.1}{1.0-j0.0} - \{(-1.0+j3.0)\times(1.02014\angle 2.605^o)\}\right.$$

$$\left. - \{(-2.0+j6.0)\times(1.03108\angle -4.831^o)\}\right]$$

$$= 1.02467 \angle -0.51^0$$

Hence

$V_1^1 = 1.04 \angle 0^0$ pu $\qquad V_2^1 = 1.02014 \angle 2.605^0$ pu

$V_3^1 = 1.03108 \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467 \angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\left[|V_1|\,(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) + |V_2|\,(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22})\right.$$

$$\left. + |V_3|\,(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}) + |V_4|\,(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24})\right]$$

$$= 1.04\,[1.04\,\{-6.0\} + 1.04\,\{11.0\} + 1.0\{-2.0\} + 1.0\,\{-3.0\}] = 0.208 \text{ pu.}$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5-j0.208}{1.04\angle 0^0} - \{(-2.0+j6.0)\times(1.04\angle 0^o)\}\right.$$

$$\left. - \{(-0.666+j2.0)\times(1.0\angle 0^o)\} - \{(-1.0+j3.0)\times(1.0\angle 0^0)\}\right]$$

$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04 \angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\ \angle 0^0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^\circ)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.04\angle 1.846^\circ)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^\circ)\}\right]$$

$$= 1.035587\ \angle -4.951^\circ \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^\circ)\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^\circ)\}\right]$$

$$= 0.9985\ \angle -0.178^\circ$$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $\qquad V_4^1 = 0.9985\ \angle -0.178^0$ pu

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \le Q_2 \le 1$ pu. If $0.25 \le Q_2 \le 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.05645\ \angle 1.849^0$ pu

$V_3^1 = 1.038546\ \angle -4.933^0$ pu $\qquad V_4^1 = 1.081446\ \angle 4.896^0$ pu

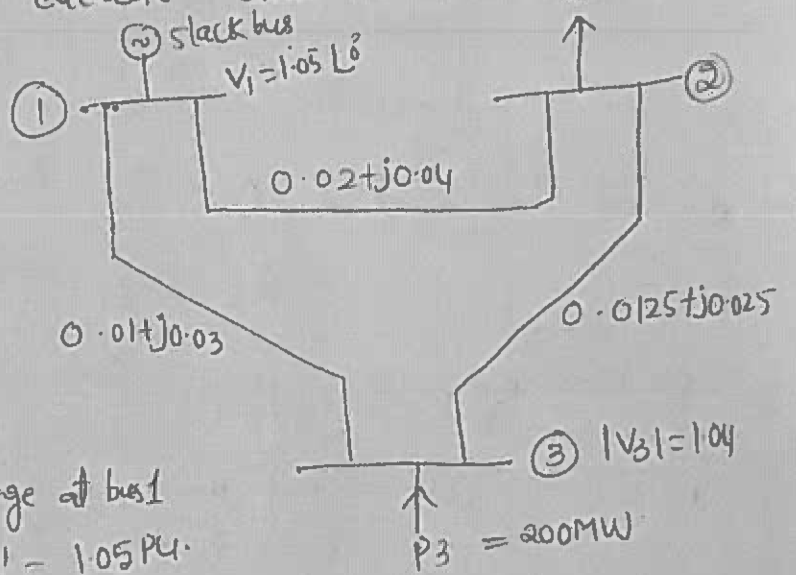
Limitations of GS load flow analysis:

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

→ Single line diagram of a simple power system, with generators at buses 1 and 3 is shown in figure. The magnitude of voltage at bus 1 is 1.05 pu. voltage magnitude at bus 3 is fixed at 1.04 pu with active power generation of 200MW. A load consisting of 400 MW and 250MVAR is taken from bus 2. line impedances are marked in pu on a 100 MVA base and the line charging susceptances are neglected. Determine the voltage at bus 2 and 3 using G.S method at the end of first iteration. Also calculate slack bus power. (400+j250) MVA



Sol →

Magnitude of voltage at bus 1 is $V_1^0 = V_1' = 1.05$ pu.

$|V_3| = 1.04$.

Base MVA = 100 MVA.

Assume a bus 2 flat voltage for PQ bus is $V_2^0 = 1 + j0.0$

Step 1 → Form Ybus matrix

$$n=3 \qquad Y_{bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} \\ -Y_{21} & Y_{22} & -Y_{23} \\ -Y_{31} & -Y_{32} & Y_{33} \end{bmatrix}_{3\times 3}$$

$$Y_{11} = Y_{12} + Y_{13} = \frac{1}{Z_{12}} + \frac{1}{Z_{13}} = \frac{1}{0.02+j0.04} + \frac{1}{0.01+j0.03}$$

$$= 20 - j50$$

$$Y_{22} = Y_{21} + Y_{23} = \frac{1}{0.02+j0.04} + \frac{1}{0.0125+j0.025} = 26 - j52$$

$$Y_{33} = Y_{31} + Y_{32} = \frac{1}{Z_{31}} + \frac{1}{Z_{32}}$$

$$= \frac{1}{0.01 + j0.03} + \frac{1}{0.0125 + j0.025} = 26 - j62.$$

$$Y_{12} = Y_{21} \Rightarrow -Y_{12} = \frac{-1}{Z_{12}} = \frac{-1}{0.02 + j0.04} = -10 + j20$$

$$Y_{13} = Y_{31} \Rightarrow -Y_{31} = -Y_{13} \Rightarrow -10 + j30$$

$$Y_{32} = Y_{23} \Rightarrow -Y_{32} = -Y_{23} = -16 + j32.$$

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j32 \end{bmatrix}.$$

Now at bus 2 (PQ) $P_2 = P_{G2} - P_{D2} = 0 - \frac{400}{100} = -4\ \text{pu}.$

$$Q_2 = Q_{G2} - Q_{D2} = 0 - \frac{250}{100} = -2.5\ \text{pu}$$

PV bus :→

$$P_3 = P_{G3} - P_{D3} = \frac{200}{100} - 0 = 2\ \text{pu}.$$

W.K.T $$V_i^{r+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k\right]$$

For first iteration $r = 0$; $i = 2$.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^0 - Y_{23}V_3^0\right] \quad V_1^0 = 1.05, \; V_3^0 = 1.04$$

$$V_2^1 = 0.9746 - j0.0423 \checkmark$$

bus 3 is PV bus a voltage controlled bus. So, reactive power is calculated as

$$Q_3^1 = -\left[V_i^* \, Y_{iq} V_q \, \sin(\theta_{ik} + \delta_k - \delta_i)\right]$$

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$= -\mathrm{Im}\,(V_3^0)^*\left[Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3\right]$

$= -\mathrm{Im}\left[(1.04)\left[(-10+j30)1.05 + (-16+j32)(0.9746-j0.0423) - (26-j62)(1.04)\right]\right]$

$= -\mathrm{Im}\left[2.392 - j1.16\right]$

$Q_3^1 = 1.16$ pu.

The voltage at bus 3 is

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3^1}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1\right]$$

$= 1.03782 - j0.005165$

$V_3^1 = 1.03783 \angle -0.285^\circ$

$\delta_3^1 = -0.285^\circ$

Now $V_3^1 = |V_3|_{specified} \angle \delta_3^1 = 1.04 \angle -0.285^\circ$

$= 1.039987 - j0.00517$

Slack bus power :→

$$S_i^* = P_i - jQ_i = V_i^* \sum_{q=1}^{n} Y_{pq}V_q$$

$i = 1, \quad n = 3.$

$P_1 - jQ_1 = V_1^*\left[Y_{11}V_1 + Y_{12}V_2^1 + Y_{13}V_3^1\right]$ $\quad V_2^1 = 0.9746 - j0.0423$

$V_3^1 = 1.039987 - j0.0517$

$P_1 - jQ_1 = 1.9488 - j1.4$ pu.

Real power(P) = $1.9488 \times 100 = 194.88$ MW.

Reactive power(Q) = $1.4 \times 100 = 140$ MVAR

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Case 1): Networking containing 1 slack bus and all are PQ buses.

Algorithem:→

i). From the given n/w calculate Ybus matrix

ii). For the load buses calculate P_i, Q_i from the given values of P_g, P_D, Q_g & Q_D → $P_i = P_G - P_D$; $Q_i = Q_G - Q_D$

iii). Assume initial values of V and δ for the load buses.

iv). Calculate V and δ by using G.S eqns.

v). Calculate the magnitude difference voltages b/w two successive iterations. if this difference is zero stop the iteration otherwise continue the iteration untill the difference is become zero. Apply the same procedure for the all load buses and calculate V and δ

vi). by applying static power flow equations calculate P and Q for the slack bus.

$$\boxed{\begin{aligned} P_i &= \sum_{k=1}^{n} |Y_{ik} V_k V_i| \cos(\theta_{ik} + \delta_k - \delta_i) \\ Q_i &= -\sum_{k=1}^{n} |Y_{ik} V_k V_i| \sin(\theta_{ik} + \delta_k - \delta_i). \end{aligned}}$$

P_i and Q_i are steady state power flow equations

Case 2: Network containing 1 slack bus 1 PV bus and all the remaining are of load buses. and the calculated value of the Q for the PV buses is within the given range of Q for the PV bus.

Algorithm:

i) Calculate Ybus for the given n/w.

ii) Assume the initial values V's for PQ bus and δ for the PV bus.

iii) Calculate Q for PV bus using static power flow eqns

$$-Q_i = \sum_{K=1}^{n} |Y_{ik} V_k V_i| \sin(\theta_{ik} + \delta_k - \delta_i)$$

iv). Check whether the calculated of Q is within the range then the PV bus. If it is with in the given range [Q_{max} to Q_{min}] then the PV bus works as PV bus and maintaining constant value of voltage.

v). Calculate δ for the PV bus using G.S eqn

$$\boxed{V_i^{q+1} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^q)^*} - \sum_{K=1}^{n} Y_{ik} V_k \right]} \Rightarrow \text{G.S eq}^n$$

vi). Calculate V and δ for the load bus using G.S eqn

vii). Calculate P and Q for the slack bus using static power flow eqns.

$$\boxed{\begin{aligned} P_i &= \sum_{K=1}^{n} |Y_{ik} V_k V_i| \cos(\theta_{ik} + \delta_k - \delta_i) \\ -Q_i &= \sum_{K=1}^{n} |Y_{ik} V_k V_i| \sin(\theta_{ik} + \delta_k - \delta_i) \end{aligned}}$$

Case 3: Network Containing 1 slack bus 1 PV bus and Q-Value for the PV bus is in the out of range for a given n/w value of Q so that PV bus is working as a PQ bus. with the value of Q for this PQ bus is the nearest value from the calculated value of Q in the given range.

Algorithm:→

i). Repeat the step 1,2,3 similar to the case 2.

ii) Check whether the calculate value is in the given range of Q, if it is not in the given range then consider PV bus as PQ bus.

iii) Repeat the steps similar to the case i

* Accelrating factor (α):→

It is used for reducing the no of iterations using G.S method

$$\boxed{V_{i\,(acc)}^{(r+1)} = V_i^{(r)} + \alpha\left[V_i^{(r+1)} - V_i^{(r)}\right]}$$

→ To speed up the operation and it is a real number.

For a suitable vale of α for trial load flows is 1.6.

→ The System data for a load -flow solutions are given (3)

below table.

The line admittances:

Bus Code	Admittance
1-2	2-j8
1-3	1-j4
2-3	0.666 - j2.664
2-4	1-j4
3-4	2-j8

The Schedule of active and reactive powers

Bus code	P	Q	V	Remarks
1	—	—	1.06	slack
2	0.5	0.2	1+j0.0	PQ
3	0.4	0.3	1+j0.0	PQ
4	0.3	0.1	1+j0.0	PQ

Determine the voltages at the end of first iteration using G.S. method. Take $\alpha = 1.6$

Sol :→ i). Determine the admittance matrix. no of buses = 4

$$Y_{bus} = \begin{bmatrix} Y_{11} & -Y_{12} & -Y_{13} & -Y_{14} \\ -Y_{21} & Y_{22} & -Y_{23} & -Y_{24} \\ -Y_{31} & -Y_{32} & Y_{33} & -Y_{34} \\ -Y_{41} & -Y_{42} & -Y_{43} & Y_{44} \end{bmatrix}_{4\times4}$$

$$Y_{11} = Y_{12} + Y_{13} = 2-j8 + 1-j4 = 3-j12$$

$$Y_{22} = Y_{21} + Y_{23} + Y_{24} = 2-j8 + 0.666 - j2.664 + 1-j4$$

$$Y_{22} = 3.666 - j14.664$$

$$Y_{33} = Y_{31} + Y_{32} + Y_{34} = 1-j4 + 0.666 - j2.664 + 2-j8$$

$$Y_{33} = 3.666 - j14.664.$$

$$Y_{44} = Y_{42} + Y_{43} = 1-j4 + 2-j8 = 3-j12.$$

Mutual admittances

$Y_{12} = Y_{21} \Rightarrow -Y_{12} = -(2-j8) = -2+j8$

$Y_{13} = Y_{31} \Rightarrow -Y_{13} = -(1-j4) = -1+j4$

$Y_{14} = Y_{41} \Rightarrow -Y_{14} = 0$

$Y_{23} = Y_{32} \Rightarrow -Y_{23} = -(0.666 - j2.664) = -0.666+j2.664$

$Y_{24} = Y_{42} \Rightarrow -Y_{24} = -(1-j4) = -1+j4$

$Y_{34} = Y_{43} \Rightarrow -Y_{34} = -(2-j8) = -2+j8$

$$Y_{bus} = \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix}$$

ii). $\xrightarrow{\text{Now}}$ Calculate P_i and Q_i

At bus 2 $P_2 = P_{G2} - P_{D2} = 0 - 0.5 = -0.5$ p.u

$Q_2 = Q_{G2} - Q_{D2} = 0 - 0.2 = -0.2$ pu

At bus 3, $P_3 = P_{G3} - P_{D3} = 0 - 0.4 = -0.4$ p.u

$Q_3 = Q_{G3} - Q_{D3} = 0 - 0.3 = -0.3$ p.u

At bus 4, $P_4 = P_{G4} - P_{D4} = 0 - 0.3 = -0.3$ pu

$Q_4 = Q_{G4} - Q_{D4} = 0 - 0.1 = -0.1$ p.u

→ iii). Assume initial values of V and δ for the load buses

Bus-1 slack bus $V_1 = 1.06$

$V_2^0 = V_3^0 = V_4^0 = 1\angle 0$

For first iteration $r = 0$

→ iv). Calculate V and δ by using G.S Gn

(4)

$$V_i^{r+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k\right]$$

Here bus (i) = 2 ; and n = 4.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.5 - j(-0.2)}{1 - j} - (2+j8)(1.06) - (-0.666 + j2.664)1 - (-1+j4).1\right]$$

$$= \frac{1}{3.666 - j14.664}\left[-0.5 + j0.2 + 2.12 - j8.48 + 0.666 - j2.664 + 1 - j4\right]$$

$$= \frac{1}{3.666 - j14.664}\left[3.286 - j14.944\right]$$

$$= (1.01187 - j0.02888) \text{ p.u.}$$

→ Calculate the magnitude difference voltage b/w two successive iterations

$$\Delta V_2^1 = V_2^1 - V_2^0 = 1.01187 - j0.02888 - (1 + j0) = 0.01187 - j0.02888 \text{ p.u.}$$

Accelerated factor voltage

$$V_2^1 acc = V_2^0 + \alpha\underbrace{\left[V_2^1 - V_2^0\right]}_{(\Delta V_2^1)}$$

$$= 1 + j0 + 1.6\,(0.01187 - j0.02888)$$

$$= 1 + 0.018992 - j0.046208 \text{ p.u.}$$

$$= 1.018992 - j0.046208 \text{ p.u.}$$

The voltage at bus 3 is

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= 0.994119 - j0.029248.$$

$$V_{3acc}^1 = V_3^0 + \alpha(V_3^1 - V_3^0)$$

$$= 1 + j0 + 1.6(0.994119 - j0.029248 - 1 - j0.0)$$

$$= 0.999059 - j0.0467968 \text{ p.u.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - \cancel{Y_{41}V_1^0} - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= 0.9716032 - j\,0.064684$$

$$V_4^0 + \alpha(V_4^1 - V_4^0)$$

$$V_{4acc}^1 = 1.0 + j0.0 + 1.6[0.9716032 - j0.064684 - 1 - j0]$$

$$V_{4acc}^1 = 0.954565 - j0.103494 \text{ p.u.}$$

→ For the above problem bus 2 is taken as a generator bus with $|V_2| = 1.04$ and reactive power constraint is $0.1 \le Q_2 \le 1.0$

Determine the voltages starting with a flat voltage profile and assuming accelerating factor as 1.0.

Sol:⇒ From the given data bus 2 is taken as a generator bus (PV)

Q_2 is not specified and $P_2 = 0.5$.

To find V_2' we first find Q_2 with $V_2 = 1.04 + j0.0$ & phase angle '0'. (5)

$$P_2 - jQ_2 = V_2^* \sum_{q=1}^{4} V_{2q} V_q \quad \simeq \quad P_i - jQ_i = \sum_{K=1}^{n} Y_{iq} V_q V_i^*$$

$$Q_2 = -\mathrm{Img}\left[V_2^{0*}(Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4\right]$$

$$= -\mathrm{Img}\Big[(1.04 - j0.0)(-2 + j8.0)(1.06) + 3.666 - j14.664(1.04)$$

$$+ (-0.666 + j2.664)(1 + j0.0) + (-1 + j4.0)1.0\Big] = 0.1108$$

Since Q_2 lies within the limits

$\therefore \quad V_2 = |V_2|$ spec.

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

Bus 2 is generator bus so P_2 and Q_2 are to be +ve.

so $Q_2 = +0.1108$.

$$V_2 = \frac{1}{3.666 - j14.664}\Big[\frac{0.5 - j0.1108}{1.04 - j0.0} - (-2 + j8.0)(1.06) - (-0.666 +$$
$$j2.664)1.0 - (-1 + j4.0)1.0\Big]$$

$V_2' = 1.0472846 + j0.0291476$;

$V_2' = 1.04 \angle 1.59°$.

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - Y_{31}V_1 - Y_{32}V_2' - Y_{34}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\Big[\frac{-0.4 + j0.3}{1 - j0.0} - (-1 + j4)1.06) - (-0.666 + 2.664)$$
$$(1.0395985 + j0.0289) - (-2 + j8)(1 + j0.0)\Big]$$

$= 0.9978866 - j0.0156070$ 5 7

Illy V_4' obtained $V_4' = 0.998065 - j022336$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{3-j12}\left[\frac{-0.3+j0.1}{1+j0.0} - 0.0\times1.06 - (-1+j4)(1.0395985+j0.02891) - (-2+j8)(0.9978866 - j0.01560705)\right]$$

$$V_4^1 = 0.998065 - j0.022336.$$

→ * For the above problem (Case1) if the reactive power constraint on generator 2 is $0.2 \le Q_2 \le 1.0$. Determine the voltages at the end of first iteration.

Sol :→ Since Q_2 calculated corresponding $V_2 = 1.04+j0.0$ is 0.1108. which is lessthan the minimum specified $Q_{2min} = 0.2$ for the given problem.

→ The reactive power generator for bus 2 is fixed at 0.2. The reactive power generator for bus 2 is load bus for this iteration. and the bus is considered as load bus P and Q are unknown voltage $V_2^0 = 1+j0.0$ for load bus and considered as -ve values.

and considered as +ve.

V and δ are +ve.

$$\therefore V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666-j14.664}\left[\frac{0.5-j0.2}{1-j0.0} - (-2+j8.0)\,1.06 - (-0.666+j0.2664) - (-1+j4.0)\,1.0\right]$$

$$V_2^1 = 1.098221 + j0.0301056$$

Illy V_3^1 and V_4^1 can be evaluated.

The no of iterations can be minimised by acceleration factor $\alpha = 1.6$

$$V_{i\,acc}^{r+1} = V_i^r + \alpha(V_i^{r+1} - V_i^r)$$

$i=2, r=0,$ $V_{2acc}^{1} = V_2^0 + 1.6\left[V_2^1 - V_2^0\right]$

$= (1+j0.0) + 1.6\left[(1.0982+j0.0301) - (1+j0)\right]$

$V_{2acc}^{1} = (1.157 + j0.048)$ p.u.

Voltage at bus-3 at the end of 1^{st} iteration $i=3, r=0.$

$$V_i^{r+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{\substack{K=1 \\ K\neq i}}^{n} Y_{ik} V_k\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 + V_{32}V_{2acc}^1 + Y_{34}V_4^0)\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4+j0.3}{(1+j0)^*} - (-1+j4)(1.06) + (-0.666+j2.664)(1.157+j0.048)(1+j0)(-2+j8)\right]$$

$$= \frac{1}{3.666 - j14.664}\left[-0.4+j0.3 - (-1.06+j0.24 - 2+j8 - 0.7705 - 0.1278 + j3.0822 - j0.03196)\right]$$

$V_3^1 = (1.019 - j0.028)$ p.u.

$i=3, r=0$ $\quad V_{3acc}^{r+1} = V_i^r + \alpha\left[V_i^{r+1} - V_i^r\right]$

$= (1+j0) + 1.6\left[(1.019 - j0.028) - (1+j0)\right]$

$V_{3acc}^{1} = 1.0304 - j0.0448$ p.u.

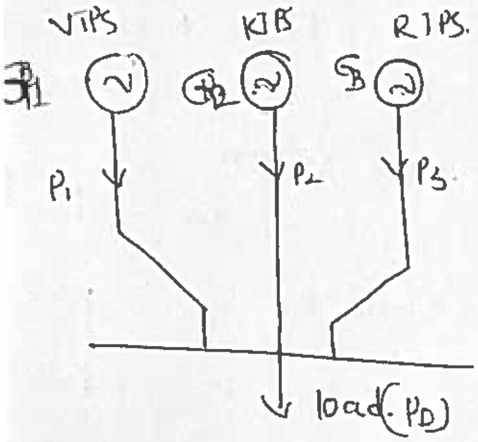
Voltage at bus 4 at the end of 1^{st} iteration $i=4, r=0.$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 + V_{42}V_{2acc}^1 + Y_{43}V_{3acc}^1\right]$$

$$= \frac{1}{3-j12}\left[\frac{-0.3-(-j0.1)}{1+j0.0} - (0.1.06 + (-1+j4)(1.157+j0.048) + (-2+j8)(1.0304 - j0.0448)\right]$$

$$= \frac{1}{3-j12}\left[-0.3+j0.1 - (-1.349+j4.58 - 1.7024 + j8.3328)\right]$$

- ECONOMIC LOAD DISPATCH -



① $P_D = P_1 + P_2 + \cdots + P_n$; $[P_L = 0]$

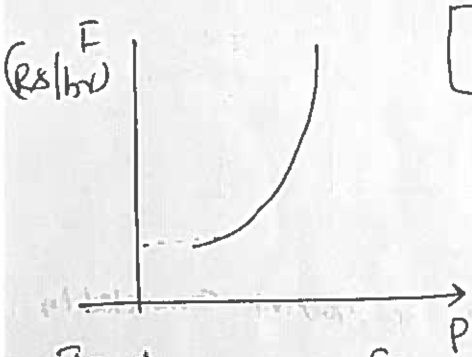
② $P_D + P_L = P_1 + P_2 + \cdots + P_n$; $[P_L \neq 0]$

condition [Constraints]

$F_T = F_1 + F_2 + \cdots + F_n \rightarrow$ Fuel consumption.

Target : objective ; cost ; min F_T

Coal $\rightarrow$ Tons/day $\rightarrow$ coal/hr $\rightarrow$ Rs/hr.

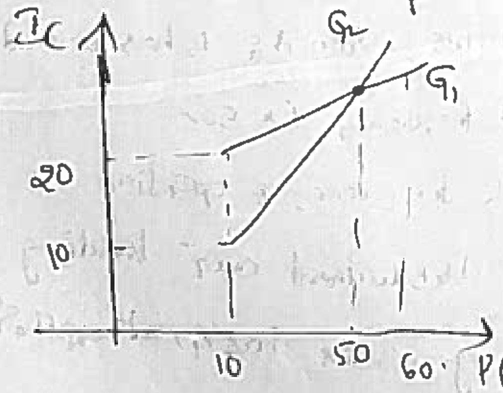


$$\boxed{F = a + bP + cP^2}$$

a, b, c constants

$$\boxed{IC = b + 2cP}$$ Rs/MWhr.

unit commitment ~~(or)~~ load sharing



→ The basic objective of economic load dispatch (ELD) is to minimise cost of the power delivered to the consumers this includes both power generation and tr. line losses.

→ For optimum economic control is each generating station satisfy

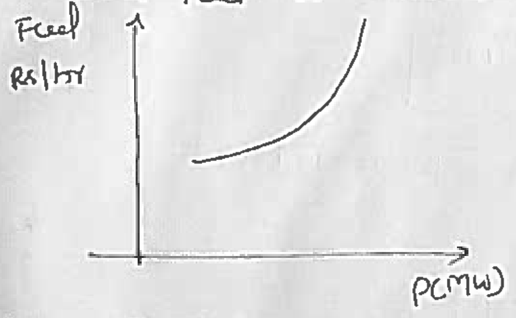
i) unit commitment.

Unit commitment problem: how much power is generating station having meeting the particular load.

→ load sheduling :- i.e, how much power has to been shared b/w the generating units for optimise the cost of the power.

Relation b/w Fuel Consumption and power :→

→ As the power generation is increasing as per the demand the consumption of the fuel like coal, gas (or) oil and Nuclear fuel is increased



$F = a + bP + cP^2$

a, b, c are constants

P = power in MW

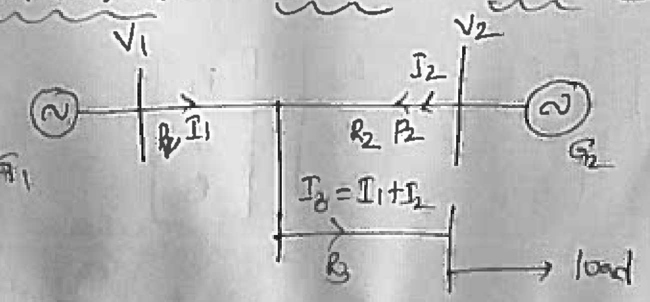
F = Fuel consumption in Rs/hr.

Assumptions :

1. Economic operation is conducted for a running cost of the generating station.

2) It is applicable to thermal, Nuclear gas station etc, fuel is utilized for producing electrical power.

→ Relation b/w tr. line loss and the power



$$I_1 = \frac{P_1}{V_1 \cos\phi_1} \; ; \; I_2 = \frac{P_2}{V_2 \cos\phi_2} \; ; \; I_3 = I_1 + I_2$$

$$= \frac{P_1}{V_1 \cos\phi_1} + \frac{P_2}{V_2 \cos\phi_2}$$

Total line loss

$$P_L = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3$$

$$= \frac{P_1^2}{V_1^2 \cos^2\phi_1} R_1 + \frac{P_2^2}{V_2^2 \cos^2\phi_2} \cdot R_2 + \left(\frac{P_1}{V_1 \cos\phi_1} + \frac{P_2}{V_2 \cos\phi_2}\right)^2 R_3$$

$$= \left[\frac{R_1 + R_3}{V_1^2 \cos^2\phi_1}\right] P_1^2 + \left[\frac{R_2 + R_3}{V_2^2 \cos^2\phi_2}\right] P_2^2 + \left[\frac{R_3}{V_1 V_2 \cos\phi_1 \cos\phi_2}\right] 2 P_1 P_2$$

$$\boxed{P_L = B_{11} P_1^2 + B_{22} P_2^2 + 2 B_{12} P_1 P_2}$$

Where B_{11}, B_{22} ; B_{12} are B coefficients.

Gen. equation:-

$$[P_L] = [P] \, [B] \, [P]^T$$

$$= \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$\boxed{B_{12} = B_{21}}$$

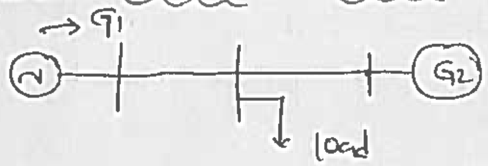
$$\boxed{P_L = B_{11} P_1^2 + B_{22} P_2^2 + 2 B_{12} P_1 P_2}$$

3 units:

$$\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

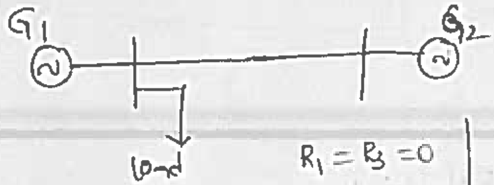
B Coefficients at different locations of loads :→

①



$R_3 = 0 \Rightarrow B_{12} = B_{21} = 0, \quad B_{11} \neq 0, \quad B_{22} \neq 0$

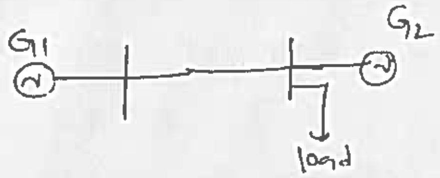
②



$R_1 = R_3 = 0$ | $B_{12} = B_{21} = B_{11} = 0$

$B_{22} \neq 0.$

③.



$R_2 = R_3 = 0$

$B_{22} = B_{12} = 0$

$B_{11} \neq 0.$

→ Two generating stations delivering a powers of $P_1 = 150$ MW $P_2 = 100$ MW, and the B coefficients are $B_{11} = 0.005, B_{22} = 0.002$ $B_{12} = B_{21} = -0.001$ find the tr. line loss?

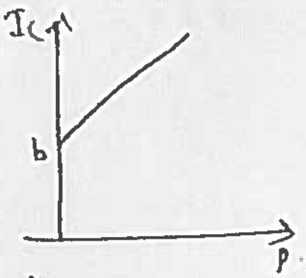
Sol:-
$$P_L = B_{11} P_1^2 + B_{22} P_2^2 + 2 B_{12} P_1 P_2$$

$$= 0.005 \times 150^2 + 100^2 \times 0.002 + 2 \times -0.01 \times 150 \times 100$$

$$= 10.25 \text{ MW}$$

Incremental cost (I.C) :→

The additional cost encured to generate one additional unit of electrical power is called I.C. measured (Rs/Mwhr).



→ $F = a + bP + CP^2$

$$\boxed{IC = \frac{dF}{dP} = b + 2CP \quad Rs/Mwhr}$$

→ Optimum condition of E.L.D :→

Case(i) : Tr. line losses are neglected:

Assume n-plants are generating power of $P_1, P_2, \ldots P_n$ supplying a to the load of P_D. - by consuming fuels of $F_1, F_2 \ldots F_n$

$F = F_1 + F_2 + \cdots + F_n$

$P_D = P_1 + P_2 + \cdots + P_n$

$\text{Min (IC)} = \frac{\partial F}{\partial P}$

Lagrangian Function :→

Main Fn = Aux. Fn + λ (Constraint)

λ = lagrangian multiplier

Main Fn (L),

Aux. Fn $= F_1 + F_2 + \cdots + F_n$

Constraint (G) $= P_D - (P_1 + P_2 + \cdots + P_n)$

$$L = F + \lambda G$$

min. cost : $\frac{\partial L}{\partial P} = 0$.

$$\frac{\partial L}{\partial P_1} = \frac{\partial}{\partial P_1}\left[F_1 + F_2 + \cdots + F_n\right] + \frac{\partial}{\partial P_1}\left[P_D - P_1 - P_2 \cdots - P_n\right] = 0.$$

P_D = constant.

$$\frac{\partial L}{\partial P_1} = \frac{\partial F_1}{\partial P_1} + \lambda(0 - 1 - 0) = 0$$

$$\frac{\partial F_1}{\partial P_1} = 0$$

Similarly for n-plant.

$$\frac{\partial F_1}{\partial P_1} = \frac{\partial F_2}{\partial P_2} = \cdots = \frac{\partial F_n}{\partial P_n} = \lambda$$

$$\boxed{(I_{C1}) = (I_{C2}) = \cdots = I_{Cn} = \lambda}$$

Case (ii): tr. lines losses are considered $(P_L \neq 0)$:

$$F = F_1 + F_2 + \cdots + F_n$$

$$(P_L + P_D) = P_1 + P_2 + \cdots + P_n$$

$$G = (P_L + P_D) - (P_1 + P_2 + \cdots + P_n).$$

$$L = F + \lambda G$$

$$= F_1 + F_2 + \cdots + F_n + \lambda\left[P_L + P_D - P_1 - P_2 \cdots - P_n\right]$$

min. cost $\frac{\partial L}{\partial P} = 0$; P_D = constant.

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2.$$

$$P_L = f(P_1, P_2 \cdots P_n).$$

$$\frac{\partial L}{\partial P_1} = \frac{\partial F_1}{\partial P_1} + \lambda\left[\frac{\partial P_L}{\partial P_1} + 0 - 1\right] = 0.$$

$$\frac{\partial F_1}{\partial P_1} = \lambda\left[1 - \frac{\partial P_L}{\partial P_1}\right] \Rightarrow \boxed{\frac{\partial F_1}{\partial P_1} \times \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \lambda}$$

$$\frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \alpha_1 = \text{penality factor.}$$

$$\frac{\partial F_1}{\partial P_1} = I_{C1}$$

Similarly $(I_C)_1\alpha_1 = (I_C)_2\alpha_2 = \cdots = (I_C)_n\alpha_n = \lambda.$

$$\alpha_n = \frac{1}{1-\frac{\partial P_L}{\partial P_n}} = \text{penality factor.}$$

→ $\frac{\partial P_L}{\partial P_n}$ = Increamental transmission ~~cost~~ Loss (ITL)

~~ITL~~ ITL:- The additional tr. losses are encurred for transmitting one additional unit of electrical power.

→

$B_{12} = B_{22} = 0$.

$P_L = B_{11} P_1^2 \quad \frac{\partial P_L}{\partial P_2} = 0$.

$$\alpha_2 = \frac{1}{1-\frac{\partial P_L}{\partial P_2}} = \frac{1}{1-0} = 1$$

→ The I.C of two generators $I_{C1} = 0.02P_1 + 60$, $I_{C2} = 0.3P_2 + 40$. and the rating of generators are 150 and 250 MW.

i). find the load sharing of each generator for a load of 200 MWs.

ii) " " savings in cost in Rs/hr for economic load condition compared to the loading proportional to the rating of generators.

(i) $P_D = P_1 + P_2 = 200\,MW$.

For optimum: $I_{C1} = I_{C2}$.

$$0.2P_1 + 60 = 0.3P_2 + 40 \quad -(2)$$

Solving (1) & (2). $P_1 = 80\,MW$

$P_2 = 120\,MW$.

(ii) In proportional to rating of generator

$P_{G1} = 150\,MW \quad P_{G2} = 250\,MW$.

$$\frac{P_{G1}}{P_{G2}} = \frac{150}{250} = 0.6$$

$P_{G_1} = 0.6 P_{G_2} \Rightarrow \quad P_1 = 0.6 P_2$

$P_1 + P_2 = 200$ — ②

$P_1 = 75\ MW, \quad P_2 = 125\ MW.$

case ②

$IC_1 = \frac{\partial F_1}{\partial P_1} = 0.2 P_1 + 60$

$F_1 = \int (0.2 P_1 + 60)\, dP_1 = 0.2 \frac{P_1^2}{2} + 60 P_1 + a_1$

$F_1 = 0.1 P_1^2 + 60 P_1 + a_1.$

$IC_2 = \frac{\partial F_2}{\partial P_2} \Rightarrow F_2 = \int 0.3 P_2 + 40.$

$= 0.3 \frac{P_2^2}{2} + 40 P_2 + a_2 \quad a_1, a_2 = \text{constants}.$

Total fuel: $F = F_1 + F_2$

$= 0.1 P_1^2 + 60 P_1 + a_1 + 0.15 P_2^2 + 40 P_2 + a_2.$

	Fuel cost
$P_1 = 80$, $P_2 = 120$	$12400 + a_1 + a_2$
$P_1 = 75$, $P_2 = 125$	$12406.9 + a_1 + a_2$

Saving Rs/hr = 6.25/-

page no: 50

Q.16.

$IC_1 = 25 + 0.2 P_{G_1}$

$IC_2 = 32 + 0.2 P_{G_2}$

$P_1 + P_2 = 250$ — ①

$IC_1 = IC_2$

$25 + 0.2\, P_{G_1} = 32 + 0.2 P_{G_2}$

solve ① & ②

$P_{G_1} = 142.5\ MW$

$P_{G_2} = 107.5\ MW$

⑰. $P_1 + P_2 = 250$

$C_1 = P_1 + 0.05 P_1^2$

$C_2 = 3 P_2 + 0.05 P_2^2.$

$\frac{dC_1}{dP_1} = IC_1 = 1 + 0.05 \times 2 P_1$

$\frac{dC_2}{\partial P_2} = IC_2 = 3 + 0.05 \times 2 P_2$

$IC_1 = IC_2$

$1 + 0.01 P_1 = 3 + 0.06 P_2$

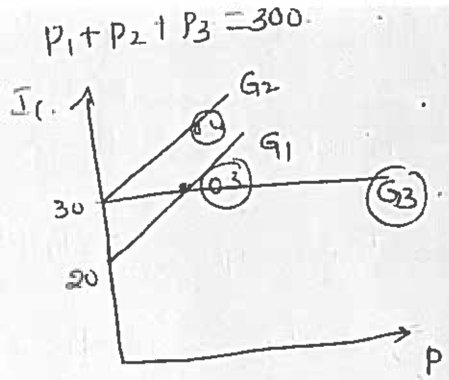
solve ① & ② $P_1 = 100\ MW$, $P_2 = 150\ MW.$

(18) $I_{C1} = 20 + 0.3P_1$ | $P_{min} = 50$

$I_{C2} = 30 + 0.4P_2$ | $P_{max} = 300$

$I_{C3} = 30$.

$P_1 + P_2 + P_3 = 300$.



$P_3 = 300$ MW

$P_1 + P_2 = 700 - 300 = 400$ MW. — (1)

$I_{C1} = I_{C2}$

$20 + 0.3P_1 = 30 + 0.4P_2$. — (2)

Solve (1) & (2). $P_1 = 242.86$

$P_2 = 157.14$.

(19).

(22). $I_C = 0.012P + 8$, $\frac{\partial P_L}{\partial P} = 0.2$, $\lambda = 25$.

$$\frac{I_C}{1 - \frac{\partial P_L}{\partial P}} = \lambda \Rightarrow \frac{0.012P + 8}{1 - 0.2} = 25 \quad P = \frac{25 \times 0.8 - 8}{0.012}$$

$$\boxed{P = 1000 \text{ MW}}$$

(23). $B_{11} = 10^{-3}$ $\quad \alpha_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P}}$ $\quad$ load is connected G_2

So $\alpha_2 = 1$.

$P_L = B_{11} P_1^2$

$\frac{\partial P_L}{\partial P} = 2B_{11}P_1 = 0.2$ $\quad = \frac{1}{1 - 0.2} = 1.25$

(24). $l \propto R$ $\quad$ P_1 in mW + losses.

$P_L = I^2 R$.

$S_1 =$

(25). $P_D = 40$ MW. $\quad P_{loss}(pu) = 0.5 P_{G1}^2 (pu)$, base $= 100$ MVA.

$I_{C1} = 10,000$ Rs/MWhr

$I_{C2} = 12,500$ Rs/MWhr.

$(I_{C1})\alpha_1 = (I_{C2})\alpha_2$

$$\alpha_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 2(0.5)P_1}$$

$\alpha_1 = \frac{1}{1-P_1}$; $\alpha_2 = 1$ ✓.

$\frac{10,000}{1-P_1} = 12,500 \times 1$ ✓. $P_1 = 0.2 pu$.

base value $(S_b) = 100$ MVA.

$P_1 = 0.2 \times 100 = 20$ MVA

$P_1 + P_2 = 40$ MW.

(a)

$P_L = 0.5 \times P_1^2$
$= 0.5 \times (0.2)^2 = 0.02 pu$

$P_L = 0.02 \times 100$
$= 2$ MVA ✓
$=$

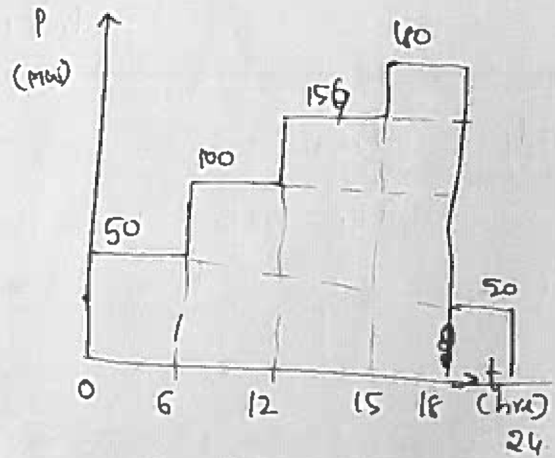
$P_D = 40$

$P_D + P_L = P_1 + P_2$

$42 = 20 + P_2 \Rightarrow P_2 = 22$ MWA ✓.

→ Two generators has I.C $\frac{dF_1}{dP_1} = IC_1 = 27.5 + 0.15P_1$; $\frac{dF_2}{dP_2} = IC_2 = 19.5 + 0.26P_2$ Rs/Mwhr. These generators operate at min & maximum of 10 and 100 MW the load curve for the station is shown in fig. find power delivered by the each generator for optimum load dispatch. and also find I.C during the each period.

Lin losses are neglected



→ Economical condition :→

$IC_1 = IC_2$

$27.5 + 0.15P_1 = 19.5 + 0.26P_2$

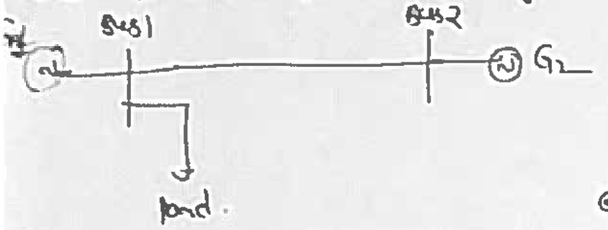
$0.26P_2 - 0.15P_1 = 8$ — ①

Time	P_D	P_1	P_2	λ
0-6 & 18-24 →	$P_1+P_2=50$ 50	12.19	37.81	=29.32 Rs/mwhr
6-12	100	43.1	56.1	34.08
12-15	150	78.14	75.86	39.22
15-18	180	94.63	85.36	41.62.

$\lambda = 27.5 + 0.15 \times 12.19$
$= 29.32$ Rs/MWhr.

→ Given a 2 bus System as shown in the fig and it is observed that when a power of 75 MW is imported to bus 1, The power loss is 5 MW find the generation needed from each plant and also power received by the load. the System λ is 20 Rs/Mwhr, The Incremental fuel Cost to the '2' plants are given by $I_{C_1} = \frac{dF_1}{dP_1} = 0.03P_1 + 15$ Rs/Mwhr

$I_{C_2} = \frac{dF_2}{dP_2} = 0.05P_2 + 18$ Rs/Mwhr.



Given $\lambda = 20$ Rs/Mwhr

Sol:- At 75MW, $P_L = 5MW$

$P_L = B_{22} P_2^2$.

$5 = B_{22}(75)^2$

$B_{22} = 8.88 \times 10^{-4}$

$(I_{C_1})\alpha_1 = (I_{C_2})\alpha_2 = \lambda$

$$\frac{I_{C_1}}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{I_{C_2}}{1 - \frac{\partial P_L}{\partial P_2}} = \lambda\ (20)$$

$\frac{\partial P_L}{\partial P_2} = 2B_{22}P_2$

$= 2 \times 8.88 \times 10^{-4}$

$$\frac{0.03P_1 + 15}{1 - 0} = \frac{0.05P_2 + 18}{1 - 2 \times 8.88 \times 10^{-4} \times P_2} = 20$$

$\boxed{P_1 = 166.66 MW}$

$P_2 = 23.38$ MW

$P_L = B_{22}P_2^2 = (23.38)^2 \times 8.88 \times 10^{-4} = 0.048$ MW.

$P_L + P_D = P_1 + P_2 = 166.66 + 23.38$

$P_D = 166.66 + 23.28 - 0.048 = 189.56$ MW.

Unit-II : Economic operations of power systems.

→ With large interconnection of electrical network, energy crisis in the world and continuous rising prices, it is essential to reduce the running charges of electrical energy i.e. reduce the fault consumption for meeting a particular load demand

→ In the load flow studies, for a particular load demand generation at all generating stations are fixed

→ In Economic load dispatch, for a particular load demand the generations at all generating stations are not fixed but allowed to vary within the specified limits with minimum fuel consumption

→ Therefore economic load dispatch problem is the solution of large no. of load flow problems and optimal solution has minimum cost of generation

System Constraints

- System Constraints
  - Equality Constraints.
  - Inequality Constraints.
    - Hard type Constraints
    - Soft type Constraints

Hard type: Constraint are specific and definite

Ex: Tap setting of ON-load tap changing transformer

Soft type: These constraints are flexible

Ex: Bus voltages and phase angles between bus voltages

Equality Constraints:

The equality Constraints are given by the basic load flow equations

$$P_p = \sum_{Q=1}^{n} e_p(e_Q G_{pQ} + f_Q B_{pQ}) + f_p(f_Q G_{pQ} - e_Q B_{pQ})$$

$$Q_p = \sum_{Q=1}^{n} f_p(e_Q G_{pQ} + f_Q B_{pQ}) - e_p(f_Q G_{pQ} - e_Q B_{pQ})$$

Where $p = 1, 2, 3, \ldots n$

e_p and f_p are real and imaginary component of voltage of node p.

G_{pQ} and B_{pQ} are nodal conductance and suseptance between nodes p and Q

Inequality Constraints :-

a) Generator Constraints :-

→ MVA loading on alternator/generator at bus-p

$$\boxed{S_p = \sqrt{P_p^2 + Q_p^2} \leq S_{sp,p}}$$ due to temperature rise conditions

→ Maximum power generation is limited by thermal consideration and minimum power generation is limited by boiler-conditions

→ If the power output of a generator per optimum operation of the system is less than a prespecified value, P_{min}. The unit is not connected to grid because it is not possible to generate that low value of power from that unit

$$\boxed{P_{p,min} < P_p \leq P_{p,max}}$$

i.e. Generated output powers cannot be outside the above range. Similarly reactive power Q_p cannot be outside the range as given by the inequality

$$\boxed{Q_{p,min} \leq Q_p \leq Q_{p,max}}$$

b) Voltage Constraints:-

→ The voltage magnitude should vary within certain limits otherwise the equipment connected to the system will not operate satisfactorily (or) additional voltage regulating devices increases the system cost

$$\boxed{\begin{gathered} |V_{p,min}| \le |V_p| \le |V_{p,max}| \\ S_{p,min} \le S_p \le S_{p,max} \end{gathered}}$$

c) Running Spare Capacity Constraints:-

→ These constraints are required to meet sudden increase in the load demand and forced outages of atleast one alternator on the system

→ Therefore, the total generation should be such that in addition to meeting the load demand and line losses a minimum spare capacity should be available

$$\boxed{P_G \ge P_{load} + P_{losses} + P_{spare}}$$

d) Transformer Tap settings

Equipment	Minimum Tap setting	Maximum Tap setting	Range of tap setting
1) Auto Transformer	0	1	$0 \le t \le 1$
2) Two winding transformer with tappings on secondary side	0	n	$0 \le t \le n$
3) Phase shifting transformer	θ_{min}	θ_{max}	$\theta_{min} \le \theta \le \theta_{max}$

e) Transmission line Constraints :

The flow of active power and reactive power through the transmission line is emitted by thermal loading capacity of the transmission line.

$$\boxed{C_p \le C_{p,max}}$$

Where C_{pmax} is maximum thermal loading capacity of line p f.) Network Security Constraints:-

→ If initially a system is operating satisfactorily and there is a sudden outage then sum of the system constraints may be violated

→ As the size of the power system network increases, the number of violated constraints increases

→ Therefore, analyse the system with outage of one branch at a time and two branches at a time

ECONOMIC LOAD DISPATCH

⇒ The basic objective of economic load dispatch (ELD) is to minimise cost of the power delivered to the consumers. This includes both power generation and tr. line losses

→ To optimum economic control is each generating station satisfy

i) Unit Committment

→ Unit Committment problem:- How much power is generating station having meeting the particular load

→ load Scheduling:- i.e. How much power has to been shared b/w the generating units for optimise the cost of the power

Relation between fuel Consumption and Power

As the power generation is increasing as per the demand the consumption of the fuels like coal, gas, oil and nuclear fuel is increased

Cost function (or) fuel Consumption :-

$$C(P) = f(P) = a + bP + cP^2$$

Cost Component 'a' is independent of power generation.

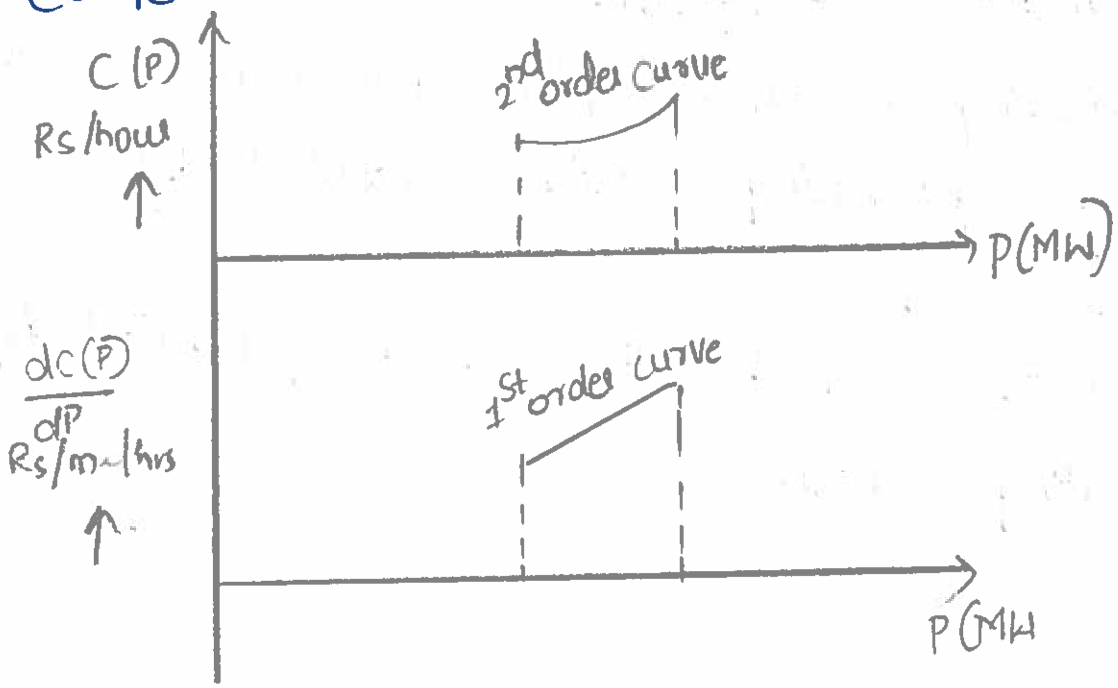
Ex:- Cost of land

Cost Component 'b' depends on power generation

Ex:- Running Charges, fuel Cost, Salaries of employees etc.

Cost Component 'c' includes depends on square of the power generated

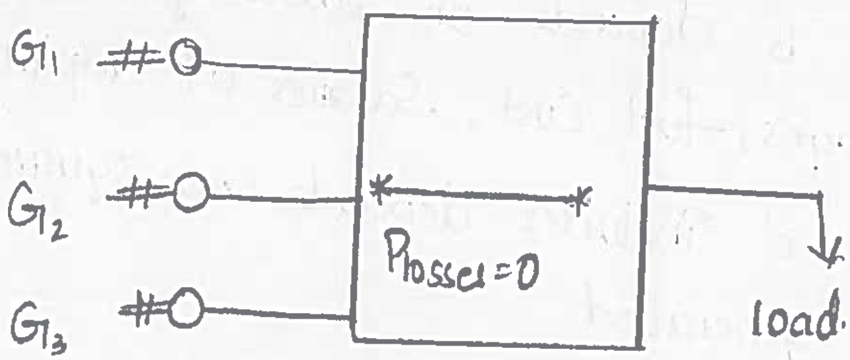
Ex:- Compensation Cost



Incremental Cost of Generation (or) Incremental Fuel Cost

$$\boxed{\frac{dC(P)}{dP} \text{ (or) } \frac{dF(P)}{dP} = b + 2cP = \alpha + \beta P \; \frac{Rs}{MW\text{-}hour}}$$

Case (i):- Economic dispatch of setting generating stations of neglecting transmission line losses



Schematic Description

→ Considering a power system network consisting of 'n' generating stations denoted by $G_1, G_2 \ldots G_n$.

→ let $P_1, P_2 \ldots P_n$ be the power generated by n generating stations

→ let P_{load} be the load denoted by Demand existing on the system

Objective:-

The objective is to minimise the total cost of generation subjected to equality constraints

Minimise $F_T = F_1(P_1) + F_2(P_2) + \ldots + F_n P(n)$

Equality Constraint, ϕ: $P_{load} = P_1 + P_2 + \ldots + P_n$

$$= \sum_{i=1}^{n} P_i.$$

$$\boxed{P_{load} - \sum_{i=1}^{n} P_i = 0.}$$

Solution:- The solution of optimisation problem is obtained by lagrange function.

$$L = F_T + \lambda \cdot \phi$$

λ = an unknown multiplier known as lagrange multiplier

$$\boxed{L = F_1(P_1) + F_2(P_2) + \ldots + F_n(P_n) + \lambda\{P_{load} - (P_1 + P_2 + \ldots P_n)\}}$$

→ lagrange function depends on $(n+1)$ variables 'n' variables corresponds to power generated by 'n' generating stations
i.e. $P_1, P_2, \ldots P_n$ and $(n+1)^{th}$ variable corresponds to unknown multiplier known as lagrange multiplier (λ)

→ The solution is obtained by differentiating lagrange function partially with respect to each of the $(n+1)$ variables one at a time and equal to zero

Differentiating lagrange function partially wrt to P_1 and equate to zero

$$\frac{\partial L}{\partial P_1} = 0 \Rightarrow \left\{ \frac{\partial F_1(P_1)}{\partial P_1} + 0 \cdots 0 \right\} + \lambda \left\{ 0 - (1+0+0 \cdots) \right\} = 0$$

$$\frac{dF_1(P_1)}{dP_1} - \lambda = 0$$

$$\boxed{\frac{dF_1(P_1)}{dP_1} = \lambda} \quad \text{——} (1)$$

Differentiating lagrange function partially with respect to P_2 and equate to zero

$$\frac{\partial L}{\partial P_2} = 0 \Rightarrow \left\{ \frac{dF_2(P_2)}{dP_2} + 0 \cdots 0 \right\} + \lambda \left\{ 0 - (0+1+0 \cdots +0) \right\} = 0$$

$$\frac{dF_2(P_2)}{dP_2} - \lambda = 0$$

$$\boxed{\frac{dF_2(P_2)}{dP_2} = \lambda} \quad \text{---} (2)$$

Differentiating lagrange function partially with respect to P_n and equate to zero

$$\frac{\partial L}{\partial P_n} = 0 \Rightarrow \left\{0 + 0 + \ldots + \frac{dF_n(P_n)}{dP_n}\right\} + \lambda\left\{0 - (0 + 0 + \ldots + 1)\right\} = 0$$

$$\frac{dF_n(P_n)}{dP_n} - \lambda = 0$$

$$\boxed{\frac{dF_n(P_n)}{dP_n} = \lambda} \quad \text{---} (n)$$

From above equations (1), (2) ... (n), the incremental cost of generations of all generating stations are equal to lagrange multiplier (λ)

$$\boxed{\frac{dF_1(P_1)}{dP_1} = \frac{dF_2(P_2)}{P_2} = \ldots\ldots = \frac{dF_n(P_n)}{P_n} = \lambda} \quad \text{---} (A)$$

Differentiating lagrange function partially with respect to λ and equate to zero

$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \{0+0+0+0+0\} + 1\{P_{load} - [P_1 + P_2 + \cdots + P_n)\} = 0$$

$$P_{load} - (P_1 + P_2 + \cdots + P_n) = 0$$

$$P_{load} - \sum_{i=1}^{n} P_i = 0$$

$$\boxed{\phi = 0} \quad \text{— (B)}$$

The equations (A) and (B) are known as coordination equations of economic load dispatch problem neglecting transmission line losses

Analysis:-

Consider two generating stations with incremental cost of generation

$$\frac{dF_1(P_1)}{dP_1} = \alpha_1 + \beta_1 P_1 \quad ; \quad \frac{dF_2(P_2)}{dP_2} = \alpha_2 + \beta_2 P_2.$$

For economic operation of power stations.

$$\frac{dF_1(P_1)}{dP_1} = \frac{dF_2(P_2)}{dP_2} = \lambda$$

$$\alpha_1 + \beta_1 P_1 = \alpha_2 + \beta_2 P_2 = \lambda$$

$$\boxed{P_1 = \frac{\lambda - \alpha_1}{\beta_1}} \quad \text{--- (1)} \qquad \boxed{P_2 = \frac{\lambda - \alpha_2}{\beta_2}} \quad \text{--- (2)}$$

Neglecting transmission line losses

$$P_1 + P_2 = P_{load}$$

$$\frac{\lambda - \alpha_1}{\beta_1} + \frac{\lambda - \alpha_2}{\beta_2} = P_{load}$$

$$\boxed{\lambda\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right) = P_{load} + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}.} \quad \text{--- (3)}$$

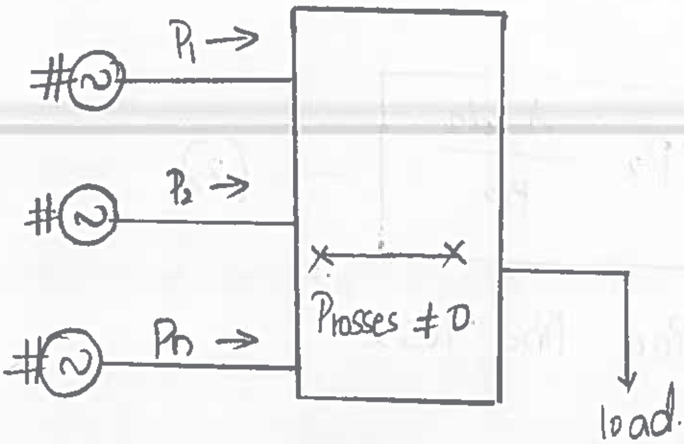
λ is calculated from equation (3) there by P_1 and P_2 are calculated from the equations (1) and (2)

For 'n' generating stations

$$\boxed{\lambda\left(\frac{1}{\beta_1} + \frac{1}{\beta_2} + \dots + \frac{1}{\beta_n}\right) = P_{load} + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \dots + \frac{\alpha_n}{\beta_n}}$$

Case 2 :- Considering transmission line losses

Schematic diagram



Schematic description :-

→ Consider a power system network consisting of 'n' generating stations, denoted by $G_1, G_2, \ldots G_n$

→ let $P_1, P_2 \ldots P_n$ be the power generated by 'n' generating stations.

→ let P_{load} be the load demand existing on the system

Objective :-

The objective is to minimize the total cost of generation considering transmission line losses subjected to equality constraints.

$$\text{Minimize } F_T = F_1(P_1) + F_2(P_2) + \ldots + F_n(P_n)$$

Equality Constraint, $\phi = P_{load} + P_{losses} = P_1 + P_2 + \dots + P_n$

$$P_{load} + P_{losses} = \sum_{i=1}^{n} P_i$$

$$\boxed{P_{load} + P_{losses} - \sum_{i=1}^{n} P_i = 0}$$

Solution:-

The solution of optimisation problem can be obtained using lagrange function

$$L = \left\{F_1(P_1) + F_2(P_2) + \dots + F_n(P_n)\right\} = F_T + \lambda\phi$$

$$+ \lambda\left\{P_{load} + P_{losses} - (P_1 + P_2 + \dots + P_n)\right\}$$

Lagrange function depends on $(n+1)$ variables where 'n' variable corresponds to power generated by 'n' generating stations $P_1, P_2 \dots P_n$ and $(n+1)^{th}$ variable is lagrange multiplier (λ)

→ The solution is obtained by differentiating lagrange function partially with respect to each of $(n+1)$ variables one at a time and equals to zero

$$\frac{\partial L}{\partial P_1} = 0 \Rightarrow \left\{\frac{dF_1(P_1)}{dP_1} + 0 + \dots + 0\right\} + \lambda\left\{0 + \frac{\partial P_{loss}}{\partial P_1} - (1 + 0 + \dots + 0)\right\}$$

$$\boxed{\frac{dF_1(P_1)}{dP_1} + \lambda\frac{\partial P_{loss}}{\partial P_1} = \lambda} \quad \text{—— (1)}$$

$$\frac{\partial L}{\partial P_2}=0 \Rightarrow \left\{0+\frac{dF_2(P_2)}{P_2}+\ldots+0\right\}+\lambda\left\{0+\frac{\partial P_{loss}}{\partial P_2}-(0+1+0\ldots+0)\right\}$$

$$\boxed{\frac{dF_2(P_2)}{P_2}+\lambda\frac{\partial P_{loss}}{\partial P_2}=\lambda} \text{ ——— } (2)$$

Differentiating lagrange function partially with respect to P_n and equals to zero

$$\frac{\partial L}{\partial P_n}=0 \Rightarrow \left\{0+0+\ldots+\frac{\partial F_n(P_n)}{dP_n}\right\}+\lambda\left\{0+\frac{\partial P_{loss}}{\partial P_n}-(0+0+\ldots 1)\right\}$$

$$\boxed{\frac{dF_n(P_n)}{P_n}+\lambda\frac{\partial P_{loss}}{\partial P_n}=\lambda} \text{ ——— } (n)$$

Differentiating lagrange function partially with respect to λ and equals to zero

$$\frac{\partial L}{\partial \lambda}=0 \Rightarrow \{0+0+0\ldots+0\}+\{P_{load}+P_{losses}-(P_1+P_2+\ldots+P_n)=0$$

$$P_{load}+P_{losses}-(P_1+P_2+\ldots+P_n)=0$$

$$\boxed{\phi=0} \text{ ——— } (n+1)$$

Equations (1), (2) . . . (n), (n+1) are known as Coordination equations by considering transmission line losses

Note:-

For ith generating station the coordination equation is

$$\boxed{\frac{dF_i(P_i)}{dP_i} + \lambda \frac{\partial P_{losses}}{\partial P_i} = \lambda}$$

Penality factor:-

→ Penality factor is a measure of fraction of transmission line losses with respect to total power generated and transmitted

→ Coordination equation for ith generating station is

$$\frac{dF_i(P_i)}{dP_i} + d \frac{\partial P_{losses}}{\partial P_i} = d$$

$$\frac{dF_i(P_i)}{dP_i} = \lambda - \lambda \frac{\partial P_{loss}}{\partial P_i}$$

$$\frac{dF_i(P_i)}{dP_i} = \lambda \left(1 - \frac{\partial P_{loss}}{\partial P_i}\right)$$

$$\lambda = \frac{1}{\left(1 - \frac{\partial P_{loss}}{\partial i}\right)} \frac{dF_i(P_i)}{dP_i}$$

$$\lambda = L_i \frac{dF_i(P_i)}{dP_i}$$

$$L_i = \frac{1}{1 - \frac{\partial P_{loss}}{\partial i}} \cong 1 + \frac{\partial P_{loss}}{\partial P_i}$$

Case 1:- Ideal transmission line.

$$P_{loss} = 0$$

$$L = 1 + \frac{\partial P_{loss}}{\partial P} = 1 + 0$$

$$\boxed{L = 1}$$

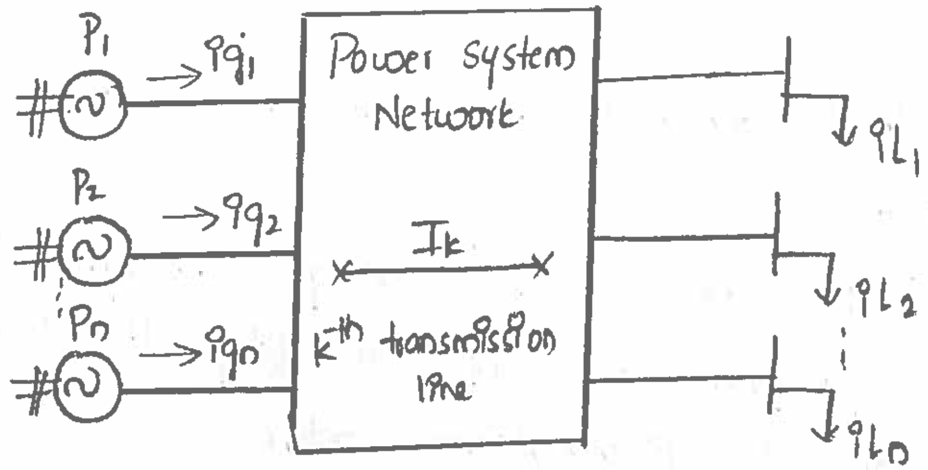
Case 2:- $\partial P_{loss} = \partial p$

$$\partial P_{loss} = \partial p$$

$$L = 1 + \frac{\partial P_{loss}}{\partial p} = 1 + 1$$

$$\boxed{L = 2}$$

Transmission loss as a function of plant generation and calculation of loss (B) Coefficients:-



Schematic Diagram

→ Consider a power system network consisting of 'n' generators and 'n' loads

→ let $P_1, P_2, \ldots P_n$ be the powers generated by 'n' generating stations $G_1, G_2, \ldots G_n$

→ let $ig_1, ig_2, \ldots ig_n$ be the 'n' generator currents

→ let $iL_1, iL_2, \ldots iL_n$ be the 'n' load currents.

3. Assumptions

→ The ratio of X and R of transmission line is same

→ The branch currents and load currents are in-phase

→ Current distribution factors are real

→ Calculation of Transmission line losses

Total load current

$$\boxed{I_L = i_{L1} + i_{L2} + \ldots\ldots + i_{Ln} = \sum_{i=1}^{n} i_{Lj}}$$

→ Assume that generator G_1 alone meets the total load demand.

let 'I_{k1}' be the current flowing through 'k^{th}' transmission line with generator G_1 alone meeting the total load demand current distribution factor

Current distribution factor, $\boxed{(d_{k1}) = \frac{I_{k1}}{I_L}}$

→ Assume that generator G_{12} alone meets the total load demand. let 'I_{k2}' be the current flowing through 'k^{th}' transmission line with generator G_2 alone meeting the total load demand

Current distribution factor $\boxed{(d_{k2}) = \frac{I_{k2}}{I_L}}$.

→ Assume the generator G_n alone meets the total load demand

let 'I_{kn}' be the current flowing through 'k^{th}'

transmission line with generator G_n alone meeting the total load demand

Current distribution factor $\boxed{(d_{kn})\ (\text{or})\ (\alpha_{kn}) = \frac{I_{kn}}{I_L}}$

In practice; 'n' generators act simultaneously to meet the total load demand

According to Kron's transformation,

Current flowing through k^{th} transmission line with 'n' generators meeting total load demand

$$\boxed{I_k = d_{k1} i_{g1} + d_{k2} i_{g2} + \dots + d_{kn} i_{gn}}$$

With three generators,

$$\boxed{I_k = d_{k1} i_{g1} + d_{k2} i_{g2} + d_{k3} i_{g3}} \quad \text{---} (1)$$

Expressing generator currents i_{g1}, i_{g2}, i_{g3} in terms of power generating P_1, P_2, P_3

Generator currents in polar form

$$\left.\begin{aligned} i_{g1} &= |i_{g1}| \angle \delta_1 = |i_{g1}| (\cos\delta_1 + j\sin\delta_1) \\ i_{g2} &= |i_{g2}| \angle \delta_2 = |i_{g2}| (\cos\delta_2 + j\sin\delta_2) \\ i_{g3} &= |i_{g3}| \angle \delta_3 = |i_{g3}| (\cos\delta_3 + j\sin\delta_3) \end{aligned}\right\} \quad \text{---} (2)$$

Substitute eqn (2) in eq (1)

$$I_k = dk_1|iq_1|(\cos\delta_1 + j\sin\delta_1) + dk_2|iq_2|(\cos\delta_2 + j\sin\delta_2) + dk_3|iq_3|(\cos\delta_3 + j\sin\delta_3)$$

$$I_k = (dk_1|iq_1|\cos\delta_1 + dk_2|iq_2|\cos\delta_2 + dk_3|iq_3|\cos\delta_3) + j(dk_1|iq_1|\sin\delta_1 + dk_2|iq_2|\sin\delta_2 + dk_3|iq_3|\sin\delta_3)$$

$$\boxed{I_k^2 = a^2 + b^2} \qquad \boxed{\therefore I_k = a + jb}$$

$$I_k^2 = dk_1^2|iq_1|^2 + dk_2^2|iq_2|^2 + dk_3^2|iq_3|^2 + 2dk_1dk_2|iq_1||iq_2|\cos\delta_1\cos\delta_2 + 2dk_2dk_3|iq_2||iq_3|\cos\delta_2\cos\delta_3 + 2dk_3dk_1|iq_3||iq_1|\cos\delta_3\cos\delta_1 + 2dk_1dk_2|iq_1||iq_2|\sin\delta_1\sin\delta_2 + 2dk_2dk_3|iq_2||iq_3|\sin\delta_2\sin\delta_3 + 2dk_3dk_1|iq_3||iq_1|\sin\delta_3\sin\delta_1$$

$$I_k^2 = dk_1^2|iq_1|^2 + dk_2^2|iq_2|^2 + dk_3^2|iq_3|^2 + 2dk_1dk_2|iq_1||iq_2|\cos(\delta_1 - \delta_2) + 2dk_2dk_3|iq_2||iq_3|\cos(\delta_2 - \delta_3) + 2dk_3dk_1|iq_3||iq_1|\cos(\delta_3 - \delta_1) \quad \text{——(3)}$$

Power generated by generating stations,

$$\left.\begin{aligned}
P_1 &= \sqrt{3}\,|V_1|\,|i_{g1}|\cos\phi_1 \text{ then } |i_{g1}| = \frac{P_1}{|V_1|\sqrt{3}\cos\phi_1} \\
P_2 &= \sqrt{3}\,|V_2|\,|i_{g2}|\cos\phi_2 \text{ then } |i_{g2}| = \frac{P_2}{\sqrt{3}\,|V_2|\cos\phi_2} \\
P_3 &= \sqrt{3}\,|V_3|\,|i_{g3}|\cos\phi_3 \text{ then } |i_{g3}| = \frac{P_3}{\sqrt{3}\,|V_3|\cos\phi_3}
\end{aligned}\right\} \text{---} (4)$$

Substitute eq^n (4) in eq (3)

$$I_k^2 = dk_1^2\left(\frac{P_1^2}{3|V_1|^2\cos^2\phi_1}\right) + dk_2^2\left(\frac{P_2^2}{3|V_2|^2\cos^2\phi_2}\right) + dk_3^2\left(\frac{P_3^2}{3|V_3|^2\cos^2\phi_3}\right)$$

$$+ 2dk_1dk_2\left(\frac{P_1P_2}{3|V_1||V_2|\cos\phi_1\cos\phi_2}\right)\cos(\delta_1-\delta_2)$$

$$+ 2dk_2dk_3\left(\frac{P_2P_3}{3|V_2||V_3|\cos\phi_2\cos\phi_3}\right)\cos(\delta_2-\delta_3)$$

$$+ 2dk_3dk_1\left(\frac{P_3P_1}{3|V_3||V_1|\cos\phi_3\cos\phi_1}\right)\cos(\delta_3-\delta_1) \text{ —— } (5)$$

Transmission line, $\boxed{P_{loss} = 3 \sum_{k=1}^{k} I_k^2 R_k}$ — (6)

Substitute equation (5) in equation (6)

$$P_{loss} = 3 \sum_{k=1}^{k} d_{k1}^2 \left(\frac{P_1^2}{3|V_1|^2 \cos^2\phi_1} \right) R_k + 3 \sum_{k=1}^{k} d_{k2}^2 \left(\frac{P_2^2}{3|V_2|^2 \cos^2\phi_2} \right) R_k +$$

$$3 \sum_{k=1}^{k} d_{k3}^2 \left(\frac{P_3^2}{3|V_3|^2 \cos^2\phi_3} \right) R_k +$$

$$3 \sum_{k=1}^{k} 2 d_{k1} d_{k2} \left(\frac{P_1 P_2}{3|V_1||V_2| \cos\phi_1 \cos\phi_2} \right) \cos(\delta_1 - \delta_2) R_k$$

$$+ 3 \sum_{k=1}^{k} 2 d_{k2} d_{k3} \left(\frac{P_2 P_3}{3|V_2||V_3| \cos\phi_2 \cos\phi_3} \right) \cos(\delta_2 - \delta_3) R_k$$

$$+ 3 \sum_{k=1}^{k} 2 d_{k3} d_{k1} \left(\frac{P_3 P_1}{3|V_3||V_1| \cos\phi_3 \cos\phi_1} \right) \cos(\delta_3 - \delta_1) R_k$$

$$P_{loss} = P_1^2 \sum_{k=1}^{k} \left(\frac{d_{k1}^2 R_k}{|V_1|^2 \cos^2\phi_1} \right) + P_2^2 \sum_{k=1}^{k} \left(\frac{d_{k2}^2 R_k}{|V_2|^2 \cos^2\phi_2} \right)$$

$$+ P_3^2 \sum_{k=1}^{k} \left(\frac{d_{k3}^2 R_k}{|V_3|^2 \cos^2\phi_3} \right) + 2P_1P_2 \sum_{k=1}^{k} \left(\frac{d_{k1} d_{k2} R_k}{|V_1||V_2| \cos\phi_1 \cos\phi_2} \right) \cos(\delta_1 - \delta_2)$$

$$+ 2P_2P_3 \sum_{k=1}^{k} \left(\frac{d_{k2} d_{k3} R_k}{|V_2||V_3| \cos\phi_2 \cos\phi_3} \right) \cos(\delta_2 - \delta_3)$$

$$+ 2P_3P_1 \sum_{k=1}^{k} \left(\frac{d_{k3} d_{k1} R_k}{|V_3||V_1| \cos\phi_3 \cos\phi_1} \right) \cos(\delta_3 - \delta_1)$$

In standard from,

$$\boxed{P_{loss} = P_1^2 B_{11} + P_2^2 B_{22} + P_3^2 B_{33} + 2P_1P_2 B_{12} + 2P_2P_3 B_{23} + 2P_3P_1 B_{31}} \quad \text{--- (7)}$$

Units of B: MW^{-1}

$$P_{loss} = P_1 B_{11} P_1 + P_2 B_{22} P_2 + P_3 B_{33} P_3 + (P_1 B_{12} P_2 + P_2 B_{21} P_1) + (P_2 B_{23} P_3 + P_3 B_{32} P_2) + (P_3 B_{31} P_1 + P_1 P_{13} P_3)$$

$B_{12} = B_{21}$; $B_{23} = B_{32}$; $B_{31} = B_{13}$

$$\boxed{P_{loss} = \sum_{i=1}^{ng=3} \sum_{j=1}^{ng=3} P_i B_{ij} P_j} \quad \text{--- (8)}$$

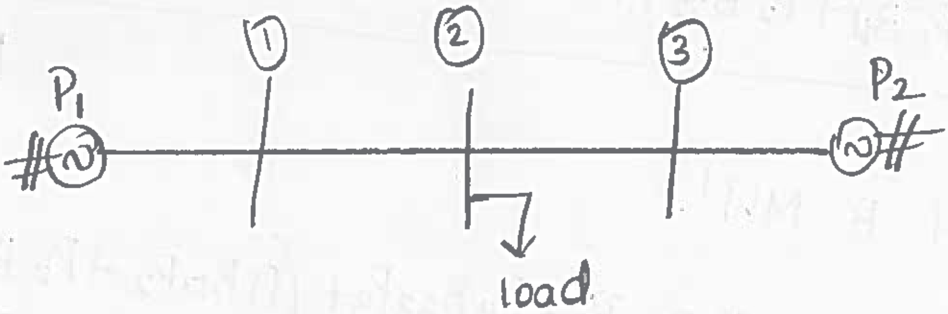
In standard form,

$$B_{mn} = \sum_{k=1}^{k} \frac{\alpha_{km}\alpha_{kn}R_k}{|V_m||V_n|\cos\phi_m \cos\phi_n} \cos(\delta_m - \delta_n)$$

$$\boxed{B_{mn} = \frac{\cos(\delta_m - \delta_n)}{|V_m||V_n|\cos\phi_m \cos\phi_n} \sum_{k=1}^{k} \alpha_{km}\alpha_{kn}R_k} \quad \text{--- (9)}$$

Calculation of Transmission line loss based on location of load in power system network.

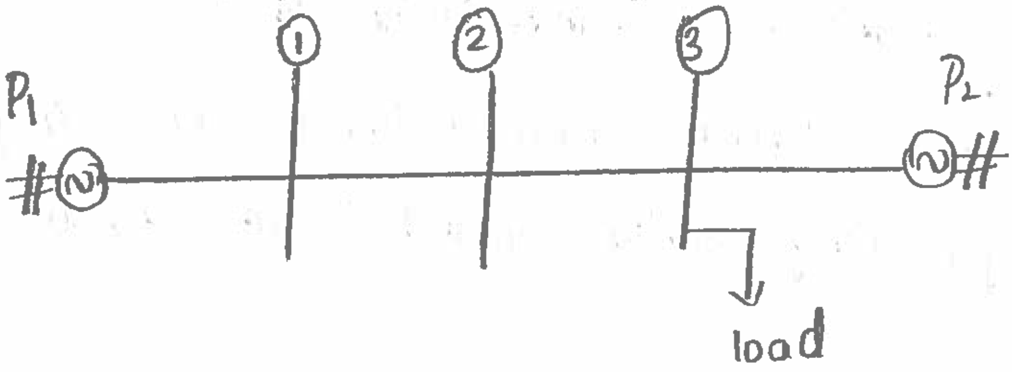
Case 1 :- load located at bus-2



$$P_{loss} = \sum_{i=1}^{2} \sum_{j=1}^{2} P_i B_{ij} P_j$$

$$\boxed{P_{loss} = \cancel{\sum_{j=1}^{2}} (P_1 B_{ij} P_1 + P_1 B_{12} P_2 + P_2 B_{21} P_1 + P_2 B_{22} P_2)}$$

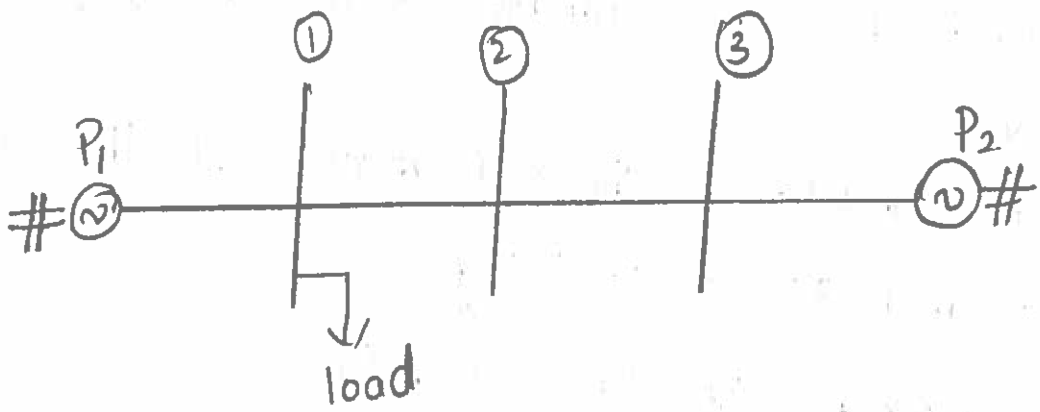
Case-2 :- load located at bus-3



$B_{22} = 0$; $B_{12} = B_{21} = 0$

$P_{loss} = P_1^2 B_{11} + 2P_1P_2B_{12} + P_2^2 B_{22}$

Case-3 :- load located at bus-1 :-



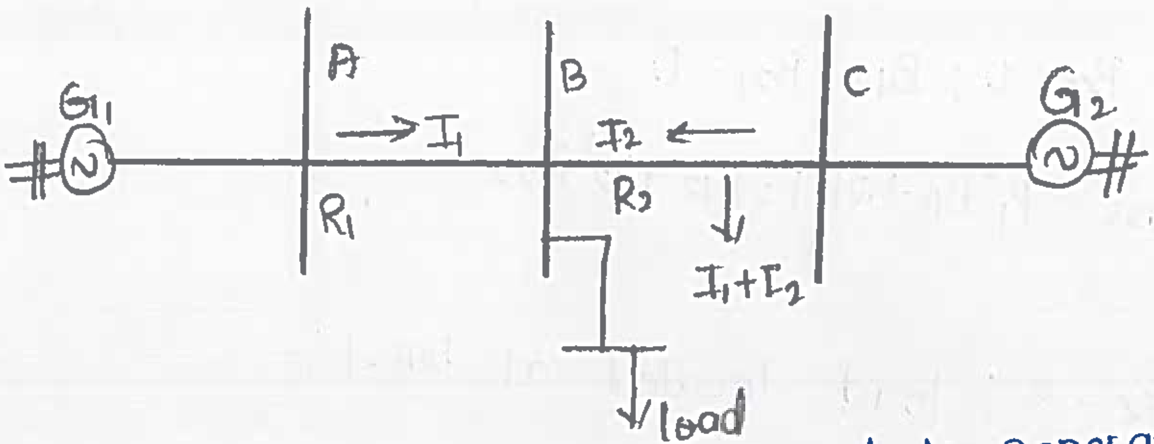
$B_{11} = 0$; $B_{12} = B_{21} = 0$

$P_{loss} = P_1^2 B_{11} + 2P_1P_2 B_{12} + P_2^2 B_{22}$

$$\boxed{P_{loss} = P_2^2 B_{22}}$$

Determination of transmission line losses in terms of resistance of transmission line:

Consider a power system network consisting of two generating stations G_1 and G_2 are shown



→ let I_1, I_2 be the currents delivered by generators G_1 & G_2

→ let R_1, R_2 and R_3 be the resistances of the sections AB, BC and BD respectively

Total transmission line losses,

$$P_{loss} = 3\left(P_{loss\,AB} + P_{loss\,BC} + P_{loss\,BD}\right)$$

$$P_{loss} = 3\left(I_1^2 R_1 + I_2^2 R_1 + (I_1+I_2)^2 R_3\right) \quad \text{—— (1)}$$

Power generated by generator-1,

$$P_1 = \sqrt{3}|V_1| I_1 \cos\phi_1 \Rightarrow I_1 = \frac{P_1}{\sqrt{3}|V_1|\cos\phi_1} \quad \text{——} (2)$$

Similarly, power generated by generator-2,

$$P_2 = \sqrt{3}|V_2| I_2 \cos\phi_2 \Rightarrow I_2 = \frac{P_2}{\sqrt{3}|V_2|\cos\phi_2} \quad \text{——} (3)$$

Substitute (2), (3) in equation (1)

$$P_{loss} = 3\left[\frac{P_1^2}{3|V_1|^2\cos^2\phi_1} R_1 + \frac{P_2^2 R_2}{3|V_2|^2\cos^2\phi_2} + \left[\frac{P_1}{\sqrt{3}V_1\cos\phi_1} + \frac{P_2}{\sqrt{3}|V_2|\cos\phi_2}\right]^2 R_3\right]$$

$$P_{loss} = \frac{P_1^2 R_1}{|V_1|^2\cos^2\phi_1} + \frac{P_2^2 R_2}{|V_2|^2\cos^2\phi_2} + 3\left(\frac{P_1^2}{3|V_1|^2\cos^2\phi_1} + \frac{P_2^2}{3|V_2|^2\cos^2\phi_1} + \frac{2P_1P_2}{3|V_1||V_2|\cos\phi_1\cos\phi_2}\right) R_3$$

$$P_{loss} = P_1^2\left(\frac{R_1+R_3}{|V_1|^2\cos^2\phi_1}\right) + P_2^2\left(\frac{R_2+R_3}{|V_2|^2\cos^2\phi_2}\right) + 2P_1P_2\left(\frac{R_3}{3|V_1||V_2|\cos\phi_1\cos\phi_2}\right) \quad \text{——} (4)$$

In standard form.

$$P_{loss} = P_1^2 B_{11} + P_2^2 B_{22} + 2P_1P_2B_{12} \quad \text{——} (5)$$

Comparing eq^n (4) and eq^n (5)

$$\boxed{B_{11} = \frac{R_1+R_3}{|V_1|^2 \cos^2\phi_1}, \quad B_{22} = \frac{R_2+R_3}{|V_2|^2\cos^2\phi_2} \\ B_{12} = \frac{R_3}{3|V_1||V_2|\cos\phi_1\cos\phi_2}}$$

A Power system consists of 2 125 MW units whose input cost data are represented by the equations $C_1 = 0.04P_1^2 + 22P_1 + 800$ Rs/hr,

$C_2 = 0.045P_2^2 + 15P_2 + 1000$ Rs/hr.

If the ~~tot~~ total recieved power 200MW Determine the load sharing between units for the most economic operation.

Sol:- $C_1 = 0.04P_1^2 + 22P_1 + 800$ Rs/hr.

$C_2 = 0.045P_2^2 + 15P_2 + 1000$ Rs/hr

$P_R = 200MW$, $P_1 = ?$, $P_2 = ?$

$$\frac{dC_1}{dP_1} = 2 \times 0.04P_1 + 22 = 0.08P_1 + 22$$

$$\frac{dC_2}{dP_2} = 2 \times 0.045P_2 + 15 = 0.09P_2 + 15$$

For economic operation,

$$\boxed{\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}}$$

$0.08P_1 + 22 = 0.09P_2 + 15$.

$0.08P_1 - 0.09P_2 = 15 - 22$.

$0.09P_2 - 0.08P_1 = 7.$

$P_1 + P_2 = 200$

$\boxed{P_2 = 200 - P_1}$

$0.09(200 - P_1) - 0.08P_1 = 7$

$18 - 0.09P_1 - 0.08P_1 = 7$

$-0.17P_1 = 7 - 18$

$P_1 = \frac{11}{0.17}$

$P_1 = 64.7\,MW$

$P_2 = 200 - P_1$

$= 200 - 64.7$

$\boxed{P_2 = 135.3\,MW}$

From the given data Generation of unit-2 is beyound the max 125MW but obtained value limit So, $P_2 = 125\,MW$ only

$P_1 = 75\,MW$ //

*) The cost functions in Rs/hr for thermal plants are given by

$$C_1 = 400 + 8.4P_1 + 0.006P_1^2$$

$$C_2 = 600 + 8.93P_2 + 0.0042P_2^2$$

$$C_3 = 650 + 6.78P_3 + 0.004P_3^2$$

where P_1, P_2, P_3 are in MW neglecting the line losses and generator limits. Determine the optimum scheduling of generation of loads

$PD = 1000$ MW

Sol: $\frac{dC_1}{dP_1}$ Given,

$$C_1 = 400 + 8.4P_1 + 0.006P_1^2$$

$$C_2 = 600 + 8.93P_2 + 0.0042P_2^2$$

$$C_3 = 650 + 6.78P_3 + 0.004P_3^2$$

$$P_D = 1000\text{MW}$$

$$\boxed{\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \frac{dC_3}{dP_3} = \lambda}$$

$$\frac{dC_1}{dP_1} = 8.4 + 2 \times 0.006P_1$$

$$= 8.4 + 0.012P_1.$$

$$= \alpha_1 + B_1 P_1$$

$\alpha_1 = 8.4$; $\beta_1 = 0.012$

$$\frac{dC_2}{dP_2} = 8.93 + 2 \times 0.0042 P_2$$

$$= 8.93 + 0.0084 P_2$$

$$= \alpha_2 + \beta_2 P_2.$$

$\alpha_2 = 8.93$; $\beta_2 = 0.0084$

$$\frac{dC_3}{dP_3} = 6.78 + 2 \times 0.004 P_3$$

$$= 6.78 + 0.008 P_3$$

$$= \alpha_3 + \beta_3 P_3$$

$\alpha_3 = 6.78$, $\beta_3 = 0.008$

$$\alpha_1 + \beta_1 P_1 = \alpha_2 + \beta_2 P_2 = \alpha_3 + \beta_3 P_3 = \lambda$$

$$P_1 = \frac{\lambda - \alpha_1}{\beta_1}, \quad P_2 = \frac{\lambda - \alpha_2}{\beta_2}, \quad P_3 = \frac{\lambda - \alpha_3}{\beta_3}$$

$$\boxed{\lambda = \frac{P_D + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3}}{\frac{1}{\beta_1} + \frac{1}{\beta_2} + \frac{1}{\beta_3}}}$$

$$= \frac{1000 + \frac{8.4}{0.012} + \frac{8.93}{0.0084} + \frac{6.78}{0.008}}{\frac{1}{0.012} + \frac{1}{0.0084} + \frac{1}{0.008}}$$

$$\lambda = \frac{1000+700+1063+847.5}{327.38}$$

$\lambda = 11.023$ Rs/Mwhr

$$P_1 = \frac{11.023-8.4}{0.012} = 218.58 \text{ MW}$$

$$P_2 = \frac{11.023-8.93}{0.0084} = 249 \text{ MW.}$$

$$P_3 = \frac{11.023-6.78}{0.008} = 530 \text{ MW.}$$

Problems:

1) The incremental cost of generations of two generating stations are $\frac{dF_1(P_1)}{dP_1} = 8+0.008P_1$, $\frac{dF_2(P_2)}{dP_2} = 6.4+0.0096P_2$. The generation limits are

$100 \leq P \leq 625$ MW.

(i) Determine the economic load dispatch schedule for a load demand of 900MW.

Sol: (i) For economic operation of power stations

$$\frac{dF_1(P_1)}{dP_2} = \frac{dF_2(P_2)}{P_2} = \lambda \frac{Rs}{hour}$$

We know that,

$$\lambda\left(\frac{1}{B_1}+\frac{1}{B_2}\right) = P_{load} + \frac{\alpha_1}{B_1} + \frac{\alpha_2}{B_2}$$

$$\lambda\left(\frac{1}{0.008}+\frac{1}{0.0096}\right) = 900 + \frac{8}{0.008} + \frac{6.4}{0.0096}$$

$$\lambda(229.1667)$$

$$\boxed{\lambda = 11.2 \text{ Rs/MW-hour}}$$

$$P_1 = \frac{\lambda-\alpha_1}{B_1} = \frac{11.2-8}{0.008} = 400\text{MW}$$

$$P_2 = \frac{\lambda-\alpha_2}{B_2} = \frac{11.2-6.4}{0.0096} = 500\text{MW}.$$

Check :- limits of generation

$100 \leq P_1 \leq 625$; $100 \leq P_2 \leq 625$

$100 \leq 400 \leq 625$; $100 \leq 500 \leq 625$

$$\boxed{\therefore P_1 = 400\,MW \quad P_2 = 500\,MW}$$

(ii) Determine the economic load dispatch schedule for a load demand of 250MW.

$$\cancel{\frac{dF_1(P_1)}{dP_1} + dF_2(P_2)}$$

$$\frac{dF_1(P_1)}{dP_1} = 8 + 0.008P_1 \quad ; \quad \frac{dF_2(P_2)}{dP_2} = 6.4 + 0.0096P_2$$

For economic operation of power stations

$$\frac{dF_1(P_1)}{dP_1} = \frac{dF_2(P_2)}{dP_2} = \lambda \frac{Rs}{MW\text{-}hour}$$

$$\lambda\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right) = P_{load} + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}$$

$$\lambda\left(\frac{1}{0.008} + \frac{1}{0.0096}\right) = 250 + \frac{8}{0.008} + \frac{6.4}{0.0096}$$

$$\boxed{\lambda = 8.38 \text{ Rs/Mw-hour.}}$$

Check: $P_1 = \frac{\lambda - \alpha_1}{\beta_1} = \frac{8.363-8}{0.008} = 45.375 \text{ MW}$

$$P_2 = \frac{\lambda - \alpha_2}{\beta_2} = \frac{8.363-6.4}{0.0096} = 204.4 \text{ MW}$$

Limits of Operation

$100 \leq P_1 \leq 625$; $100 \leq P_2 \leq 625$

$100 \leq 45 \leq 625$; $100 \leq 205 \leq 625$

Set $P_1 = P_{1,min} = 100$

$P_1 + P_2 = P_{load}$

$100 + P_2 = 250$

$$\boxed{P_2 = 150 \text{ MW}}$$

(iii) Determine the economic load dispatch schedule with total load demand of 1200MW.

For economic operation of power stations

$$\frac{dF_1(P_1)}{dP_1} + \frac{dF_2(P_2)}{dP_2} = \lambda \quad \text{Rs/Mw-hour}$$

$$\lambda\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right) = P_{load} + \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}$$

$$\lambda\left(\frac{1}{0.008}+\frac{1}{0.0096}\right)=1200+\frac{8}{0.008}+\frac{6.4}{0.0096}$$

$$\boxed{\lambda = 12.5\,MW}$$

Check :- $P_1 = \frac{\lambda - \alpha_1}{\beta_1} = \frac{12.5-8}{0.008} = 562.5\,MW.$

$P_2 = \frac{\lambda - \alpha_2}{\beta_2} = \frac{12.5-6.4}{0.0096} = 635.4\,MW$

limits of operation

$100 \le P_1 \le 625$; $100 \le P_2 \le 625$

$100 \le 562.5 \le 625$; $100 \le 635.4 \le 625$

Set $P_2 = P_{2,max} = 625\,MW.$

$P_1 + P_2 = P_{load}$

$P_1 + 625 = 1200$

$P_1 = 575$

$$\boxed{\begin{array}{l} P_1 = 575\,MW \\ P_2 = 625\,MW. \end{array}}$$

(ii) Determine the minimum load demand from which four generating stations operate economically

For economic operation of generating stations

$$\frac{dF_1(P_1)}{dP_1} = \frac{dF_2(P_2)}{dP_2} = \lambda \quad Rs/MW\text{-hour.}$$

$$P_{1min} = \frac{\lambda_{1,min} - \alpha_1}{\beta_1}$$

$$100 = \frac{\lambda_{1,min} - 8}{0.008}$$

$$\boxed{\lambda_{1min} = 8.8 \quad Rs/MW\text{-hour.}}$$

$$P_{2,min} = \frac{\lambda_{2,min} - \alpha_2}{\beta_2}$$

$$100 = \frac{\lambda_{2,min} - 6.4}{0.0096}$$

$$\boxed{\lambda_{2,min} = 7.36 \quad Rs/MW\text{-hour}}$$

Select $\lambda_{min} = \lambda_{1,min} = 8.8$ Rs/MW-hour. [To get both λ value +ve even though 7.36 is less]

$$P_{1,min} = \frac{\lambda_{1,min} - \alpha_1}{\beta_1} = \frac{8.8 - 8}{0.008} = 100\,MW.$$

$$P_{2,min} = \frac{\lambda_{min} - \alpha_1}{\beta_1} = \frac{8.8 - 6.4}{0.0096} = 250\,MW.$$

$$P_{load,min} = P_{1,min} + P_{2,min}$$

$$= 100 + 250$$

$$\boxed{P_{load,min} = 350\,MW}$$

[P_1 and P_2 share load economically from a min load demand of 350MW

(v) Determine the maximum load demand upto which P_1 and P_2 continue to share the load economically

$$\frac{dF_1(P_1)}{d(P_1)} = \frac{dF_2(P_2)}{dP_2} = \lambda$$

$$P_{1,max} = \frac{\lambda_{1max} - \alpha_1}{\beta_1}$$

$$625 = \frac{\lambda_{1max} - 8}{0.08}$$

$$\boxed{\lambda_{1,max} = 13 \quad Rs/MW\text{-}hour}$$

$$P_{2,max} = \frac{\lambda_{2max} - \alpha_2}{\beta_2}$$

$$625 = \frac{\lambda_{2max} - 8.4}{0.0096}$$

$$\boxed{\lambda_2 max = 12.4 \ \text{RS/MW-hour}}$$

Select $\lambda max = \lambda_2 max = 12.4$ RS/MW-hour

$$P_1 max = \frac{\lambda max - \alpha_1}{\beta_1} = \frac{12.4-8}{0.08} = 550 \text{ MW}$$

$$P_2 max = \frac{\lambda max - \alpha_2}{\beta_2} = \frac{12.4-6.4}{0.0096} = 625 \text{MW}$$

$$P_{load, max} = P_1 max + P_2 max.$$

$$= 550 + 625$$

$$\boxed{P_{load}, max = 1175 \text{MW}.}$$

i.e. P_1 and P_2 share the load demand economically till a maximum load demand of 1175MW is reached.

2 150 MW, 220MW, 220 MW are the ratings of 3 units located in a thermal power station their respective incremental cost on

$\frac{dC_1}{dP_1} = 0.11P_1+12$; $\frac{dC_2}{dP_2} = 0.1P_2+13$; $\frac{dC_3}{dP_3} = 0.095P_3+14$

where P_1, P_2 and P_3 are the generations in MW. Determine the economic load allocation between the 3 units. When the total demand is (i) 350MW (ii) 500MW.

(i) Total load demand = 350MW

For economic operation of power sys stations, incremental cost of generation of all units generating stations must be equal.

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \frac{dC_3}{dP_3} = \lambda$$

$$0.11P_1+12 = 0.1P_2+13 = 0.095P_3+14 = \lambda$$

$\therefore \boxed{P_1 = \frac{\lambda-12}{0.11}}$ $\boxed{P_2 = \frac{\lambda-13}{0.1}}$ $\boxed{P_3 = \frac{\lambda-14}{0.095}}$

Finding λ:-

$$\lambda\left(\frac{1}{\beta_1}+\frac{1}{B_2}+\frac{1}{B_3}\right)=P_D+\left(\frac{\alpha_1}{B_1}+\frac{\alpha_2}{B_2}+\frac{\alpha_3}{B_3}\right)$$

$$\lambda\left(\frac{1}{0.11}+\frac{1}{0.1}+\frac{1}{0.095}\right)=350+\left(\frac{12}{0.11}+\frac{13}{0.1}+\frac{14}{0.095}\right)$$

$$\boxed{\lambda = 24.86 \text{ Rs/MW-hour.}}$$

$$P_1=\frac{24.86-12}{0.11}=116.5\text{ MW.}$$

$$P_2=\frac{\lambda-13}{0.1}=118.6\text{ MW.}$$

$$P_3=\frac{\lambda-14}{0.095}=\frac{24.86-14}{0.095}=113.89\text{ MW.}$$

Verifying the limits of generation

$0\le P_1\le 150$ $\quad$ $0\le P_2\le 220$ $\quad$ $0\le P_3\le 220$

$0\le 116.5\le 150$ $\quad$ $0\le 118.6\le 220$ $\quad$ $0\le 113.89\le 220$

$$\boxed{\begin{aligned}P_1&=116.5\text{ MW}\\P_2&=118.6\text{ MW}\\P_3&=113.89\text{ MW}\end{aligned}}$$

(iii) Total demand = 500MW.

For economic operation, incremental cost of generations of all units in a generating stations must be equal.

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \frac{dC_3}{dP_3} = \lambda$$

$$0.11P_1 + 12 = 0.1P_2 + 13 = 0.095P_3 + 14 = \lambda$$

$$P_1 = \frac{\lambda - 12}{0.11}, \quad P_2 = \frac{\lambda - 13}{0.1}, \quad P_3 = \frac{\lambda - 14}{}$$

$$\lambda\left(\frac{1}{\beta_1} + \frac{1}{\beta_2} + \frac{1}{\beta_3}\right) = P_D + \left(\frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3}\right)$$

$$\lambda\left(\frac{1}{0.11} + \frac{1}{0.1} + \frac{1}{0.095}\right) = 500 + \left(\frac{12}{0.11} + \frac{13}{0.1} + \frac{14}{0.095}\right)$$

$$\boxed{\lambda = 29.93 \text{ Rs/MW hour}}$$

$$P_1 = \frac{\lambda - 12}{0.11} = \frac{29.93 - 12}{0.11} = 163 \text{ MW}$$

$$P_2 = \frac{\lambda - 13}{0.1} = \frac{29.93 - 13}{0.1} = 169.3 \text{ MW}$$

$$P_3 = \frac{\lambda - 14}{0.095} = \frac{29.93 - 14}{0.095} = 167.68 \text{ MW}$$

Verifying the limits of generation:

$0 \le P_1 \le 150$ $\quad$ $0 \le P_2 \le 220$ $\quad$ $0 \le P_3 \le 220$

$0 \le 163 \le 150$ $\quad$ $0 \le 169.3 \le 220$ $\quad$ $0 \le 167.68$ MW

Set $\boxed{P_1 = P_{1,max} = 150MW}$

As P_1 violates maximum limit of generation

set $P_1 = P_{1,max}$

$$P_D = P_1 + P_2 + P_3$$

$$500 = 150 + P_2 + P_3$$

$$\boxed{P_2 + P_3 = 350\,MW}$$

Finding λ :-

$$\lambda\left(\frac{1}{\beta_2} + \frac{1}{\beta_3}\right) = P_D + \left(\frac{\alpha_2}{\beta_2} + \frac{\alpha_3}{\beta_3}\right)$$

$$\lambda\left(\frac{1}{0.1} + \frac{1}{0.095}\right) = 350 + \left(\frac{13}{0.1} + \frac{14}{0.095}\right)$$

$$\boxed{\lambda = 30.56 \text{ Rs/MWhour}}$$

$$P_2 = \frac{\lambda - 13}{0.1} = \frac{30.56 - 13}{0.1} = 175.6\,MW$$

$$P_3 = \frac{\lambda - 14}{0.095} = \frac{30.56 - 14}{0.095} = 174.31 MW$$

Verifying the limits of generation

$0 \le P_2 \le 220$

$0 \le 175.6 \le 220$

$0 \le P_3 \le 220$

$0 \le 174.31$ MW

$$\boxed{P_1 = 150 MW \quad , P_2 = 175.6 MW \quad , P_3 = 174.31 MW}$$

Incremental fuel cost in Rs/MW-hour for two units in are given by $\frac{dF_1}{dP_1} = 0.1P_1 + 20$ Rs/MW-hour, $\frac{dF_2}{dP_2} = 0.12P_2 + 16$ Rs/MW-hour, The minimum and maximum loads on each unit are 20MW and 125MW respectively.

(i) Determine the incremental fuel cost and allocation of loads between the units for minimum cost in Rs/hour

(ii) Determine the savings of fuel cost for a total load demand for economic distribution of 200MW.

Sol: For economic operation, incremental cost of generations of all both units in a generatings station are equal.

$$\frac{dF_1}{dP_1} = 0.1P_1 + 20 \quad ; \quad \frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \lambda$$

$$0.1P_1 + 20 = 0.12P_2 + 16 = \lambda$$

$$0.1P_1 - 0.12P_2 = -4$$

$$10P_1 - 12P_2 = -400 \quad \text{—— (1)}$$

WKT, $P_1 + P_2 = \text{Load}$

$$\boxed{P_1 + P_2 = 200\,MW} \quad \text{—— (2)}$$

From equations (1) & (2),

$P_1 = 90.9\,MW$

$P_2 = 109.09\,MW$.

Verifying the limits.

$20 \leq P_1 \leq 125$

$20 \leq 90.91 \leq 125$

$20 \leq P_2\ (109.09) \leq 125$

$20 \leq 109.09 \leq 125$

$\therefore\ P_1 = 90.91\ MW$

$P_2 = 109.09\,MW$.

(ii) Cost function of unit-1

$$F_1(P_1) = \int \left(\frac{dF_1(P_1)}{dP_1}\right) dP_1 = \int (0.1P_1 + 20)\, dP_1 = \frac{0.1P_1^2}{2} + 20P_1 + k_1$$

$$\boxed{F_1(P_1) = 0.05P_1^2 + 20P_1}$$

Cost function of unit-2

$$F_2(P_2) = \int \left(\frac{dF_2(P_2)}{dP_2}\right) dP_2 = \int (0.12P_2 + 16)\, dP_2 = \frac{0.12P_2^2}{2} + 16P_2 + k_2$$

$$\boxed{F_2(P_2) = 0.06P_2^2 + 16P_2}$$

Cost of Generating

$$F_{cost} = F_1(P_1) + F_2(P_2) = (0.05P_1^2 + 20P_1) + (0.06P_2^2 + 16P_2)$$

Total Cost based on equal load distribution,

$$F_{equal\ load} = (0.05P_1^2 + 20P_1) + (0.06P_2^2 + 16P_2)$$

$$= (0.05(100)^2 + 20(100)] + [0.06(100)^2 + 16(100))$$

$$F_{equal\ load} = 47100 \text{ Rs/hour.}$$

$$\boxed{F_{equal\ load} = 4700 \text{ Rs/hour}}$$

Total Cost based on economic dispatch

$F_{economic\ dispatch} = (0.05P_1^2 + 20P_1) + (0.06P_2^2 + 16P_2)$

$= (0.05 \times 90.91^2 + 20 \times 190.91) + (0.06 \times 109.09^2 + 16 \times 109.09)^2$

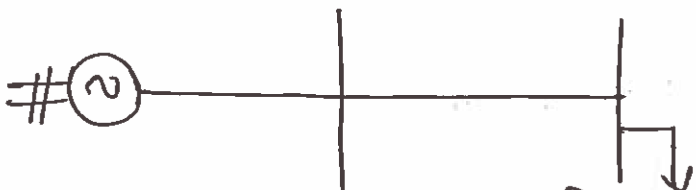
$\boxed{F_{economic\ dispatch} = 4690.91 \text{ Rs/hour}}$

Savings $= 4700 - 4690.91$

$\boxed{\text{Savings} = 9.09 \text{ Rs/hour}}$

Determine the incremental cost of recieved power and penalty factor of the plant given below.
The incremental cost of production is

$\frac{dC_1}{dP_{G1}} = 0.1P_{G1} + 3$ Rs/MWhour



$P_G = 10MW$ $\quad$ $P_D = 8MW$

$P_{loss} = P_{G1} - P_D = 10 - 8$

$\boxed{P_{loss} = 2MW}$

$$L = \frac{1}{1 - \frac{\partial P_{loss}}{\partial P_G}} = \frac{1}{1 - \frac{2}{10}}$$

$$\boxed{L = 1.25}$$

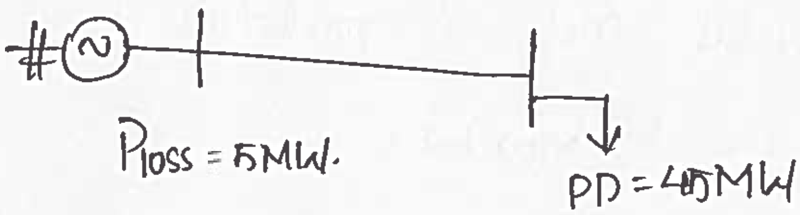
Incremental Cost $(\lambda) = L \times \frac{dc}{dP_G}$

$$= L \times (0.1 P_G + 3)$$

$$= 1.25 \times (0.1 \times 10 + 3)$$

$$\boxed{\lambda = 5 \text{ Rs/MW-hour}}$$

2) The penality factor for the figure given below is



$P_{loss} = P_G - P_D$

From $P_G = P_{loss} + P_D = 5 + 45$

$$\boxed{P_G = 50 MW}$$

$$L = \frac{1}{1 - \frac{\partial P_{loss}}{P_G}} = \frac{1}{1 - \frac{5}{50}}$$

$$\boxed{L = 1.11}$$

3)

In a two plant system load is located at plant-2. For a change in load of 5MW, the change in generation at plant-1 is 8MW. Determine the penality factor

$$\partial P_{G11} = 8MW, \quad \partial P_D = 5MW$$

$$\partial P_{loss} = \partial P_{G11} - \partial P_D = 8 - 5$$

$$\boxed{\partial P_{loss} = 3MW}$$

$$L \text{ at } G_{11} = L_1 = \frac{1}{1 - \frac{\partial P_{loss}}{\partial P_{G1}}} = \frac{1}{1 - \frac{3}{8}}$$

$$\boxed{L_1 = 1.6}$$

As G_{12} and load are connected to same bus

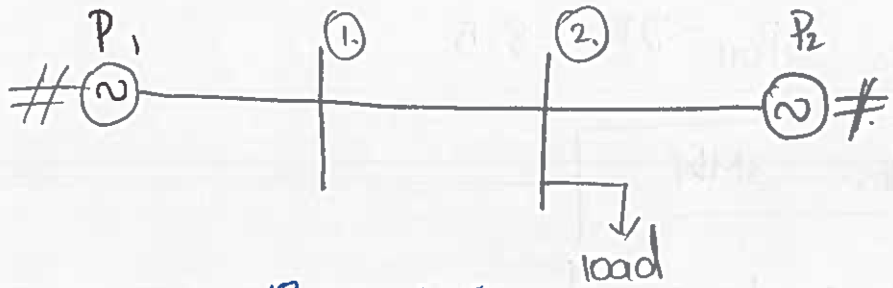
$$\partial P_{loss} = 0$$

$$L_2 = \frac{1}{1 - \frac{\partial P_{loss}}{P_{G12}}} = \frac{1}{1 - \frac{0}{\partial P_{G2}}}$$

$$\boxed{L_2 = 1}$$

A power system consists of 2 plants connected to a tie line and load is located at plant-2. A 100 MW is transmitted from plant-1, a loss of 10 MW occurs in the transmission line. Determine the generation schedule at both plants and power received by the load when 'λ' for the system is Rs. 25 /MW-hr and incremental fuel cost are given by

$\frac{dF_1}{dP_1} = 0.03P_1 + 17$ Rs/MW-hour, $\frac{dF_2}{dP_2} = 0.06P_2 + 19$ Rs/MW-hr



With two generating stations

$$P_{loss} = P_1^2 B_{11} + 2P_1 P_2 B_{12} + P_2^2 B_{22}$$

As load is connected to bus-2 to which generator-2 is also connected

$$B_{22} = 0, \quad B_{12} = B_{21} = 0$$

$$P_{loss} = P_1^2 B_{11}$$

$$10 = (100)^2 B_{11}$$

$$\boxed{B_{11} = 10^{-3} MW^{-1}}$$

Coordination equation for plant - 1

$$\frac{dF_1(P_1)}{dP_1} + \lambda \frac{\partial P_{loss}}{\partial P_1} = \lambda$$

$$0.03P_1 + 17 + \lambda \frac{\partial}{\partial P_1}(P_1^2 B_{11}) + \lambda$$

$$0.03P_1 + 17 + \lambda \times 2P_1 B_{11} = \lambda$$

$$0.03P_1 + 17 + 25 \times 2 \times P_1 (10^{-3}) = 25$$

$$\boxed{P_1 = 100 MW}$$

Coordination Equation for plant - 2

$$\frac{dF_2(P_2)}{dP_2} + \lambda \frac{\partial P_{loss}}{\partial P_2} = \lambda$$

$$0.06P_2 + 19 + \lambda \frac{\partial}{\partial P_2}(P_1^2 B_{11}) = \lambda$$

$$0.06P_2 + 19 = 25$$

$$\boxed{P_2 = 100 MW}$$

Transmission line losses, $P_{losses} = P_1^2 B_{11}$

$$= (100)^2 (10^{-3})$$

$$\boxed{P_{losses} = 10 MW}$$

Total load demand = Generation - losses

$$P_D = P_1 + P_2 - P_{losses}$$

$$= 100 + 100 - 10$$

$$\boxed{P_D = 190 \text{ MW}}$$

If the power recieved by the load is 200MW, Determine the savings in Rs/hour obtain by coordinating rather than by considering line losses but not coordinating with losses considered but not coordinated.

$$\frac{dF_1(P_1)}{dP_1} = \frac{dF_2(P_2)}{dP_2}$$

$$0.03P_1 + 17 = 0.06P_2 + 19$$

$$P_2 = \frac{0.03P_1 - 2}{0.06} \quad \text{— (1)}$$

WKT, Considering the losses

$$P_1 + P_2 - P_{losses} = P_D$$

$$P_1 + \left(\frac{0.03P_1 - 2}{0.06}\right) - \left(P_1^2 \times 10^{-3}\right) = 200$$

$$1.5P_1 - 10^{-3}P_1^2 = \frac{700}{3}$$

$$\boxed{P_1 = 176.26 \text{ MW}}$$

From eq (i)

$$\boxed{P_2 = 54.8 MW}$$

Cost function of plant-1

$$F_1(P_1) = \int \frac{dF_1(P_1)}{dP_1} \cdot dP_1 = \int (0.03P_1 + 17)\, dP_1$$

$$= 0.03 \frac{P_1^2}{2} + 17P_1 + k_1$$

$$\boxed{F_1(P_1) = 0.015P_1^2 + 17P_1}$$

Cost function of plant-2

$$F_2(P_2) = \int \frac{dF_2(P_2)}{dP_2} \cdot dP_2 = \int (0.06P_2 + 19)\, dP_2$$

$$= 0.06 \frac{P_2^2}{2} + 19P_2 + k_2$$

$$\boxed{F_2(P_2) = 0.03P_2^2 + 19P_2}$$

Floss - Considered but not Coordinated $= F_1(P_1)\Big|_{P_1=176.26} + F_2(P_2)\Big|_{P_2=54.8}$

$$= 0.015(176.26)^2 + 17(176.26) + 0.03(54.8)^2 + 19(54.8)$$

Floss → Considered but not Considered $= 4593.27$ Rs/hour.

$F_{losses} \to$ Coordinated $= F_1(P_1)\Big|_{P_1=100} + F_2(P_2)\Big|_{P_2=100}$

$$= 0.015P_1^2 + 17P_1 + 0.03P_2^2 + 19P_2$$

$$= 0.015(100)^2 + 17(100) + 0.03(100)^2 + 19(100)$$

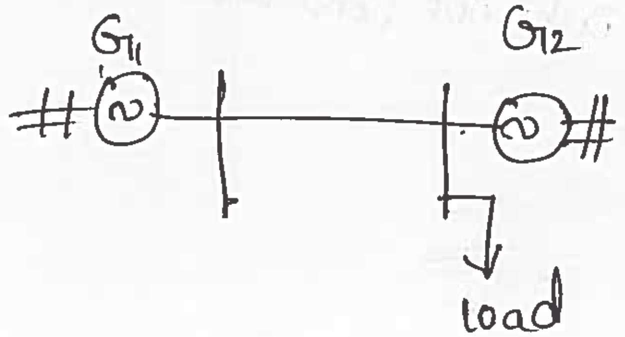
$\boxed{F_{losses} \to \text{Coordinated} = 4050 \text{ Rs/hour.}}$

Savings in Rs/hour $= 4593.27 - 4050$

$\boxed{\text{Savings in Rs/hr} = 543.27 \text{ Rs/hour}}$

A two bus System is shown in the fig. If a load of 125MW is transmitted from plant-1 to load. The losses are 15.625MW. Determine the generation Schedule and load demand if the Cost of recieved power is 24Rs/MWhour. Solve by coordination equation method and penalty factor method. The incremental production Cost of the plant are:-

$$\frac{dF_1(P_1)}{dP_1} = 0.025P_1 + 15, \quad \frac{dF_2(P_2)}{dP_2} = 0.05P_2 + 20$$



Sol: As load and generator G_2 are connected

$B_{22}=0$; $B_{12}=B_{21}=0$

For a two-bus system, transmission line losses

$$P_{loss} = P_1^2 B_{11} + 2P_1P_2B_{12} + P_2^2 B_{22}$$

$$\boxed{P_{loss} = P_1^2 B_{11}}$$

Given $P_{loss} = 15.625$, $P_1 = 125$

$$15.625 = (125)^2 B_{11}$$

$$\boxed{B_{11} = 0.001 = 10^{-3} MW^{-1}}$$

M-1:- Using Coordination equations

Coordination equation for plant-1,

$$\frac{dF_1(P_1)}{dP_1} + \lambda \frac{\partial P_{loss}}{\partial P_1} = \lambda$$

$$(0.025P_1 + 15) + \lambda \frac{\partial}{\partial P_1}(P_1^2 B_{11}) = \lambda$$

$$(0.025P_1 + 15) + 2\lambda P_1 B_{11} = \lambda$$

$$(0.025P_1 + 15) + 2 \times 24 \times P_1 \times 10^{-3} = 24$$

$$\boxed{P_1 = 123.28 MW}$$

Coordination equation for plant-2.

$$\frac{dF_2(P_2)}{dP_2} + \lambda \frac{\partial P_{loss}}{\partial P_2} = \lambda$$

$$(0.05 P_2 + 20) + \lambda \frac{\partial}{\partial P_2}(P_1^2 B_{11}) = 24$$

$$\boxed{P_2 = 80 MW}$$

Total Generation $(P_G) = P_1 + P_2 = 123.28 + 80$

$$\boxed{P_G = 203.28 \text{ MW}}$$

$$P_{loss} = P_1^2 B_{11} = (123.28)^2 \times 10^{-3}$$

$$\boxed{P_{loss} = 15.19 MW}$$

$$P_{load} = P_1 + P_2 - P_{loss}$$

$$= 203.28 - 15.19$$

$$\boxed{P_{load} = 188.09 MW}$$

Method - 2 : Penality factor

Penality factor at plant -1 ;

$$L_1 = \frac{1}{1 - \frac{\partial P_{loss}}{\partial P_1}}$$

$$= \frac{1}{1 - \frac{\partial}{\partial P_1}(P_1^2 B_{11})}$$

$$= \frac{1}{1 - 2 P_1 B_{11}}$$

$$= \frac{1}{1-2P_1B_{11}}$$

$$L_1 = \frac{1}{1-2P_1B_{11}} = \frac{1}{1-2P_1 \times 10^{-3}}$$

Coordination equation considering penalty factor

$$\frac{dF_1(P_1)}{dP_1} \times L_1 = \lambda$$

$$(0.025P_1 + 15) \times \frac{1}{1-0.002P_1} = 24$$

$$\boxed{P_1 = 123.28\,MW}$$

Penalty factor of plant-2.

$$L_2 = \frac{1}{1-\frac{\partial P_{loss}}{\partial P_2}} = \frac{1}{1-0} = 1$$

$$\boxed{L_2 = 1}$$

Considering the considering equation with penalty factor,

$$\frac{dF_2(P_2)}{dP_2} \times L_2 = \lambda$$

$$(0.05P_2 + 20) \times 1 = 24$$

$$\boxed{P_2 = 80\,MW}$$

Total Generation (P_G) $= P_1 + P_2 = 123.28 + 80$

$$P_G = 203.28 \text{ MW}$$

$P_{loss} = P_1^2 B_{11} = (123.28)^2 \times 10^{-3}$

$$\boxed{P_{loss} = 15.19 \text{ MW}}$$

load Demand $(P_{load}) = P_G - P_{loss}$

$$\Rightarrow 203.28 - 15.19$$

$$\boxed{P_{load} = 188.09 \text{ MW}}$$

For the power system network given in the figure voltage of Bus-B is $1\angle 0^\circ$ pu. The current flowing in the sections AB and CB are $1.05\angle 0^\circ$ pu and $0.9\angle 0^\circ$ pu. The transmission line impedances are $Z_{AB} = 0.05 + j0.2$ pu, $Z_{BC} = 0.04 + j0.16$ pu, $Z_{BD} = 0.03 + j0.12$ pu. Determine the transmission line loss coefficients and total transmission line losses in pu.

sol: Voltage at Bus-A $= |V_1|$

Voltage at Bus-C $= |V_2|$

Now, $V_1 - V_B = V_{AB}$

$$V_p = V_{AB} + V_B = I_1 Z_{AB} + V_B$$

$$= 1.05\angle 0^\circ (0.05 + j0.2) + 1\angle 0^\circ$$

$V_1 = 0.0525 + j0.21 + 1$

$$\boxed{V_1 = 1.0525 + j0.21\,pu}$$

$$\boxed{V_1 \cos\phi_1 = 1.0525\,pu}$$

Consider, $V_2 - V_B = V_{BC}$

$V_2 = V_{BC} + V_B$

$= I_2 Z_{BC} + V_B$

$= 0.9\angle 0° \times (0.04 + j0.16) + 1\angle 0°$

$$\boxed{V_2 = 1.036 + j0.144\,pu}$$

$$\boxed{V_2 \cos\phi_2 = 1.036\,pu}$$

$$B_{11} = \frac{R_{AB} + R_{BD}}{|V_1|^2 \cos^2\phi_1} = \frac{0.05 + 0.03}{(1.05^2)^2}$$

$$\boxed{B_{11} = 0.072\,pu}$$

$$B_{22} = \frac{R_{BC} + R_{BD}}{|V_2|^2 \cos^2\phi_2} = \frac{0.04 + 0.03}{(1.036)^2}$$

$$\boxed{B_{22} = 0.065\,pu}$$

$$B_{12} = \frac{RBD}{|V_1||V_2| \cos\phi_1 \cos\phi_2} = 0.027\ pu$$

Complex power generated by generator G_1,

$$S_1 = V_1 I_1^* = (1.052 + j0.21)(1.05\angle 0°)^*$$

$$\boxed{S_1 = 1.127\angle 11.27°\ pu}$$

$$P_1 = S_1 \cos\phi_1 = 1.127 \cos(11.27) = 1.105\ pu$$

Complex power generated by generator G_2,

$$S_2 = V_2 I_2^* = (1.036 + j0.14)(0.9\angle 0°) = 0.9324 + j0.126$$

$$\boxed{S_2 = 0.94\angle 7.69°\ pu}$$

$$P_2 = S_2 \cos\phi_2 = 0.94 \cos(7.64) = 0.931\ pu.$$

$$P_{loss} = P_1^2 B_{11} + P_2^2 B_{22} + 2P_1P_2B_{12}$$

$$\boxed{P_{loss} = 0.2\ pu}$$

Q. The Incremental fuel costs in Rs/Ms-h for 2-plants are given by $\frac{dF_1}{dP_1} = 0.03P_1 + 16$; $\frac{dF_2}{dP_2} = 0.05P_2 + 12$ This loss co-eff of the system are given by

$B_{11} = 0.005$, $B_{12} = 0.0012$ and $B_{22} = 0.002$

The load to be met is 190MW, determine the economic operating schedule and the corresponding cost generation if the tr. lines are included but not coordinated.

$$\frac{dF_1}{dP_1} = 0.03P_1 + 16$$

$$\frac{dF_2}{dP_2} = 0.05P_2 + 12$$

The optimum load schedule is given by

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$0.03P_1 + 16 = 0.05P_2 + 12$$

$$0.03P_1 - 0.05P_2 = 12 - 16$$

$$\boxed{0.03P_1 - 0.05P_2 = -4}$$

$$0.05P_2 - 0.03P_1 = 4$$

$$0.05P_2 = 4 + 0.03P_1$$

$$P_2 = \frac{4 + 0.03P_1}{0.05}$$

$$\boxed{P_2 = 80 + 0.6P_1} \quad \text{— (1)}$$

Power loss (P_L) = $B_{11}P_1^2 + 2B_{12}P_1P_2 + B_{22}P_2^2$

$$P_L = 0.005P_1^2 + 2(-0.0012)P_1P_2 + 0.002P_2^2$$

$$\boxed{P_L = 0.005P_1^2 - 0.0024P_1P_2 + 0.002P_2^2}$$

Power delivered to the load is given by:

$$P_1 + P_2 = P_L + P_D$$

$$P_1 + P_2 - P_L = P_D$$

$$P_1 + P_2 - 0.005P_1^2 + 0.0024P_1P_2 - 0.002P_2^2 = 190$$

sub eq ① in the eqn

$$P_1 + 80 + 0.6P_1 - 0.005P_1^2 + 0.0024P_1(80 + 0.6P_1) - 0.002(80 + 0.6P_1)^2 = 190$$

$$\downarrow$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$-0.0043P_1^2 + 16P_1 - 122.8 = 0$$

$$ax^2 + bx + c = 0$$

$\therefore$ $P_1 = 263.86\,MW$; $P_1 = 108.23\,MW$

Given $P_D = 190\,MW$

$P_1 = 108.23$; $P_2 = 80 + 0.6(108.23)$

$$P_2 = 144.93\,MW.$$

$$\frac{dF_1}{dP_1} = 0.03P_1 + 16$$

Integrating on B.S

$$F_1 = \frac{0.03P_1^2}{2} + 16P_1$$

$$\boxed{F_1 = 1907.38\ Rs}$$

$$\frac{dF_2}{dP_2} = 0.05P_2 + 12$$

Integrating on B.S

$$F_2 = \frac{0.05P_2^2}{2} + 12P_2$$

$$= \frac{0.05(144.93)^2}{2} + 12(144.93)$$

$$\boxed{F_2 = 2264.27\ Rs}$$

Total Fuel cost $(F_T) = F_1 + F_2$

$$= 1907.38 + 2264.27$$

$$\boxed{F_T = 4171.65\ Rs}$$

Q. A 2-Bus system is shown in figure if a load of 125MW is transmitted from the land 1 to load a loss of 15.625 MW is occured, determine the generation schedule and the load Demand if the cost of received power is 24Rs/MW-h solving the problem using coordinated eqn and the penalty factor method approach the incremental fuel cost are given by

$$\frac{dF_1}{dP_1} = 0.025P_1 + 15$$

$$\frac{dF_2}{dP_2} = 0.05P_2 + 20$$

load is connected at BUS-2

$$B_{11} \neq 0 \ ; \ B_{12} = B_{21} = B_{22} = 0$$

$$P_L = B_{11} P_1^2$$

$$15.625 = B_{11} P_1^2$$

$$15.625 = B_{11} \times (125)^2$$

$$\boxed{B_{11} = 0.001}$$

co-ordinated eqn

$$\frac{dF_1}{dP_1} + \lambda \frac{dF_2}{dP_2} = \lambda$$

$$0.025P_1 + 15 + \lambda \frac{d}{dP}[B_{11}P_1^2] = \lambda$$

$$0.025P_1 + 15 + \lambda(0.002P_1) = \lambda$$

$$0.025P_1 + 15 = \lambda(1 - 0.002P_1)$$

$$0.025P_1 + 15 = 24(1 - 0.002P_1)$$

$$0.025P_1 + 15 = 24 - 0.048P_1$$

$$0.025P_1 + 0.048P_1 = 24 - 15$$

$$0.073 P_1 = 19$$

$$\boxed{P_1 = 123.28\,MW}$$

for plant-2:

$$\frac{dF_2}{dP_2} = \lambda$$

$$0.05P_2 + 20 = 24$$

$$\boxed{P_2 = 80\,MW}$$

$$P_L = B_{11} P_1^2$$

$$= 0.001 \times (123.28)^2$$

$$\boxed{P_L = 15.19\,MW}$$

$$P_D = P_1 + P_2 - P_L$$

$$= 123.38 + 80 - 15.19$$

$$\boxed{P_D = 188.1\,MW}$$

→ Distribution of Load between plants :→

The load on power system varies w.r.to the time and is never constant. The minimum demand that always exist on the system is called Base-load and the power station supplying This load is called as Base load plant. The peak demands of the load at any point above the base load is called peak load and the power station supplying this load is called peak load station & plant.

The following advantages for the co-ordinating two plants are given below.

i). Different power plants have different operating costs, efficiencies By interconnection economic loading can be achieved and overall efficiency increases. Thereby energy can be supplied to consumers at lower cost.

ii). Inter connected power system reduces overall installed capacity of the requirements.

→

→ In the event of fault at one station, the power can be supplied from the other plant.

→ During rainy seasons the hydro stations are loaded fully and thermal stations are lightly. Wastage of water during rainy season can be avoided by interconnection b/w Hydro and thermal plants. During summer season the hydro power can be minimised and Thermal power raised.

→ Generating units of higher unit capacity can be installed and operated economically.

→ $$\text{load factor} = \frac{\text{Avg load}}{\text{Max demand}}$$

$$\text{loss factor} = \frac{\text{Avg power loss}}{\text{power loss at peak load}}$$

→ loss formula coefficients are obtained by the applications interconnected power system n/w in which no of sources are same but the no of load are reduced to one equivalent load. It is called B_{ij} coefficients

→ penality factor: To maintain the optimal allocation of power generating station with minimum cost power delievered to the consumers.

$$\text{penality Factor } (\alpha) = \frac{\partial F_1}{\partial P_1} \times \frac{1}{1 - \frac{\partial P_L}{\partial P_1}}$$

off diagonal Elements

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}$$

$$\frac{\partial P_2}{\partial e_2} = e_2 G_{23} - f_2 B_{23}$$

$$= -1.666$$

$$\frac{\partial P_3}{\partial e_3} = -1.666$$

$$\frac{\partial P_p}{\partial f_p} = e_p B_{pq} + f_p G_{pq}$$

$$\frac{\partial P_2}{\partial f_2} = -5.0$$

$$\frac{\partial P_3}{\partial f_3} = -5.0$$

Similarly we find out the partial derivation of the reactive Diagonal Element.

$$\frac{\partial Q_p}{\partial e_p} = 2 e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\frac{\partial Q_2}{\partial e_2} = 8.525 \quad ;$$

$$\frac{\partial Q_3}{\partial e_3} = 19.1$$

$$\frac{\partial Q_p}{\partial f_p} = 2 f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial Q_2}{\partial f_2} = -2.991$$

$$\frac{\partial Q_3}{\partial f_3} = -6.966$$

Similarly

$$\frac{\partial Q_2}{\partial f_2} =$$

$$\frac{\partial Q_3}{\partial f_3} =$$

$$\frac{\partial Q_2}{\partial f_3} ; \frac{\partial Q_3}{\partial f_2}$$

The set of linear equations-1 are

$$\begin{bmatrix} 0.275 \\ -0.3 \\ 0.225 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 2.846 & -1.666 & 8.975 & -5.0 \\ -1.666 & 6.366 & -5.0 & 20.90 \\ 8.525 & -5 & -2.991 & 1.666 \\ -5.0 & 19.1 & 1.666 & -6.966 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \frac{\partial P_2}{\partial f_3} & \frac{\partial P_3}{\partial f_3} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} \\ \frac{\partial Q_3}{\partial e_3} & \frac{\partial Q_3}{\partial e_3} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} \end{bmatrix}$$

UNIT-III POWER FACTOR CONTROL

Introduction. load fr problem- megawatt frequency Control channel, mVAR voltages Control channel - Dynamic interaction between p-f and Q-v loops. Mathematical model of Speed governing systems - Turbine models. derision of power system into control areas: p-f Control of Single Control area (uncontrolled & Controlled) p-f Control of two area Systems (Oncontrolled & Controlled).

Introduction:

The wide spread use of Electric clocks, the need for satisfactory operation of power stations running in parallel and the relation between System frequency and the speed of the motors has led to the requirement of close regulation of power system frequency. Since the Control of System frequency and load depends upon the governers of the prime movers we must understand governer operation.

⇒ When Power generation Equals load demand [$P_G = P_D$] System frequency remains stable.

⇒ When Sudden increse in load (P_D) this Causes the generator to slow down because more power is being drawn than supplied.

⇒ When Generator slows down, the System frequency drops.

i.e $\boxed{F \propto N}$ f - frequency
N - speed

⇒ The Speed governer detects the drop in Speed (N) / fr (f)

⇒ It increases the fuel input (or) water flow to the turbine

⇒ This increases mechanical power input to the generator.

Turbine Response

→ With increases input, the generator produce more power.

→ This helps the generator speed up and frequency begins to recover

steady-state deviation.

The frequency does not return exactly to the original value

It settles slightly lower- this is called steady-state frequency deviation.

This is shown by the governor characteristic curve in the figure.

Governer characteristics

→ The slop of the line shows how much fr drops per increase in load

→ A small frequency drop is allowed to avoid overaction of the system.

Role of LFC (Load frequency control)

⇒ load frequency control automatically adjust generation to bring frequency closer to nominal

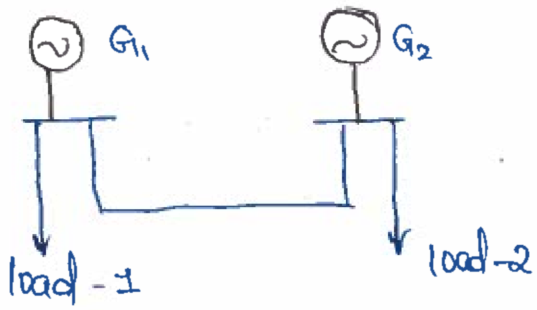
⇒ which helps share load among generators and maintain grid stability

→ Especially important in multi-area power systems.

Load frequency problem

If the system consist of a single machine connected to a group of loads the speed and frequency change in accordance with the governer characteristics as the load changes

There are 2-stations S_1 & S_2 inter connected through tie-line



Case i: Flat frequency regulation

- Assume that the increse in the load demand its all the bus 1/bus-2
- Assume that G_1 meets increse in load demand to have constant fr
- This method of regulating Generation G_1 for the changes in load demand to have constant fr is known as flat frequency regulation
- Generator G_2 delivers rated output and is operating at base load.

Case ii: Parallel frequency regulation.

The possibility of sharing the change in load such that generator G_1 & G_2 regulate their generations to maintain frequency constant is known as parallel frequency regulation.

Case iii: Flat Tie-line loading

This method of regulating the generation to have constant fr such that the change in the load in a particular area is taken care by generating stations of that particular area only there by ensuring constant tie-line loading

Case iv : Tie-line load bias Control

⇒ This method is most commonly implemented in real time

→ The interconnected power system network regulates fr irrespective of where the fr deviation has originated.

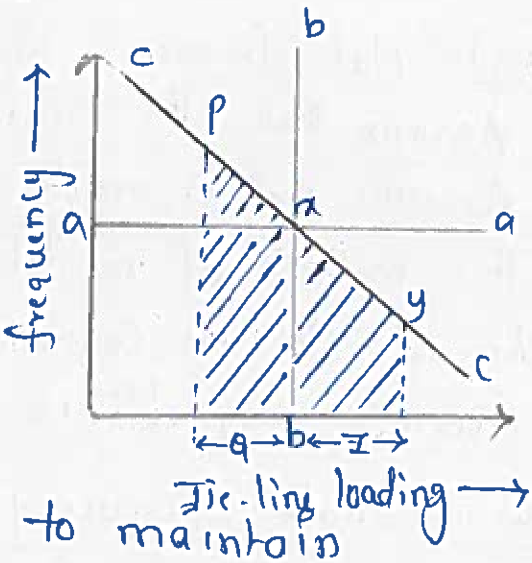
→ The Equipment Consist of master load frequency controller and tie-line recorded.

Tie-line recorded biases the load frequency controller by changing the control point until the desired relationship is established between system frequency and tie-line loading.

→ The characteristics 'aa' represents constant frequency

→ If generating station is trying to obtain constant fr then the tie-line loading varies b/w min & max limits.

→ The characteristics 'bb' represents constant tie-line loading



⇒ If the power system network is trying to maintain constant tie-line loading then the frequency varies between minimum and maximum limit.

→ In real time, the operating point lies along the characteristics 'cc'

Case 1 Increse in load demand

→ let 'x' be the initial operating frequency
→ As load demand increses, frequency decreses.
→ let y be the new frequency
→ to list out the frequency back to 'x' the additional power supply through the line equal to 'z'

Case 2 Decrese in load Demand

- Let x be the initial operating frequency
- As load demand decreses frequency increses

let 'p' be the new operating frequency
The depict power to be Transmitted through the line is Q.

Mega-Watt frequency Control and MVAR- voltage Control

Significance:

→ The load demand in a power System networks is never Constant but the objectives is to maintain Constant frequency
- Constant fr Can be maintained if ~~there~~ is a balance between generation and load demand.
- The change in frequency due to change in load demand is known as frequency Error (Δf).

Frequency Error is an index of mismatch between generation and load Demand.

The changes in the active power dependent on changes in load angle and independent of bus voltage magnitude.

- The changes in reactive power ΔQ_i depends on changes in bus voltage magnitude Control (or) P-F Control.

Load frequency Control (or) P-F Control

Objective:

The objective of P-F Control (or) load-frequency Control is to to Control frequency and Simultaneously Control active power transmitted through the tie-line.

Frequency Sensor Senses the change in frequency and gives frequency error as output.

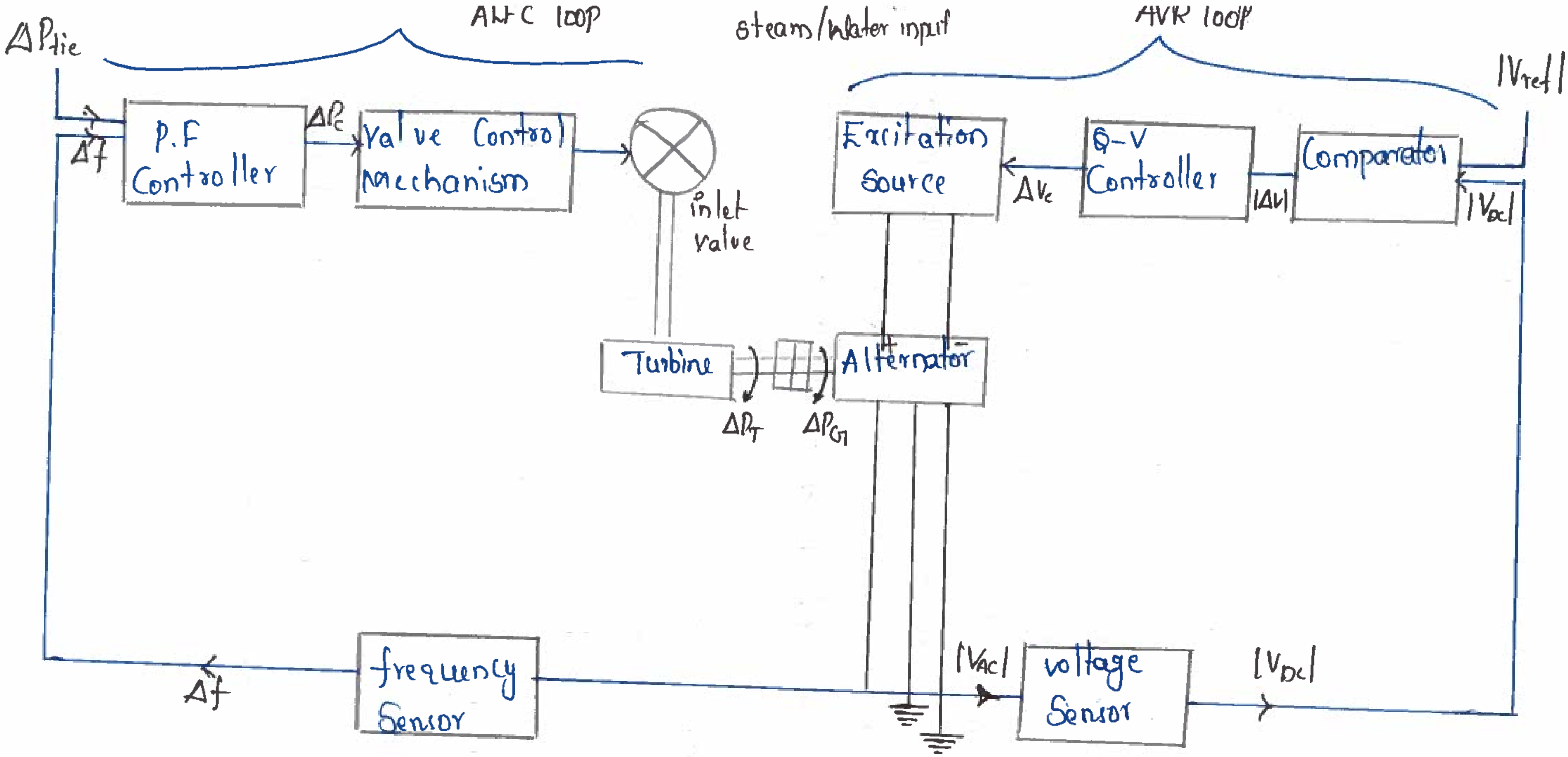
P-F Controller Senses the change in frequency i.e frequency error Δf and increment in tie-line active power.

These two Sensor Signals (Δf and ΔP_{tie}) are amplitude mixed and transform and P-F Controller gives the output as active power command signal (ΔP_c).

Active power Command Signal is fed as input to value - Control mechanism.

The value Control mechanism changes the steam input latter input to the turbine Such that generation Equal Equal to Load demand.

P-F Control Can be Considered as Automatic load frequency Control



ΔP_{tie}
ALFC loop
steam/water input
AVR loop
$|V_{ref}|$
Δf
P.F Controller
ΔP_c
Valve Control Mechanism
inlet valve
Excitation Source
ΔV_c
Q–V Controller
$|\Delta V|$
Comparator
$|V_{DC}|$
Turbine
Alternator
ΔP_T
ΔP_G
frequency Sensor
Δf
$|V_{AC}|$
voltage Sensor
$|V_{DC}|$

Excitation voltage Control (or) Q-V Control:

- The objective of Q-V control (or) Excitation voltage control is to control generated terminal voltage.

voltage Sensor senses generator terminal AC voltage and converts into equivalent DC voltage.

The equivalent DC voltage is compared with reference with reference voltage by comparator.

- The output of comparator is voltage Error 'ΔV' and fed as input to Q-V controller.

The output of Q-V controller is the reactive power command signal ΔQ_c and is fed as input to excitation source.

The excitation source changes field current such that voltage error is equal to 'zero' is obtained.

Q-V control can be considered as Automatic voltage regulation (AVR).

Dynamic Interaction between P-f Control and Q-V Control

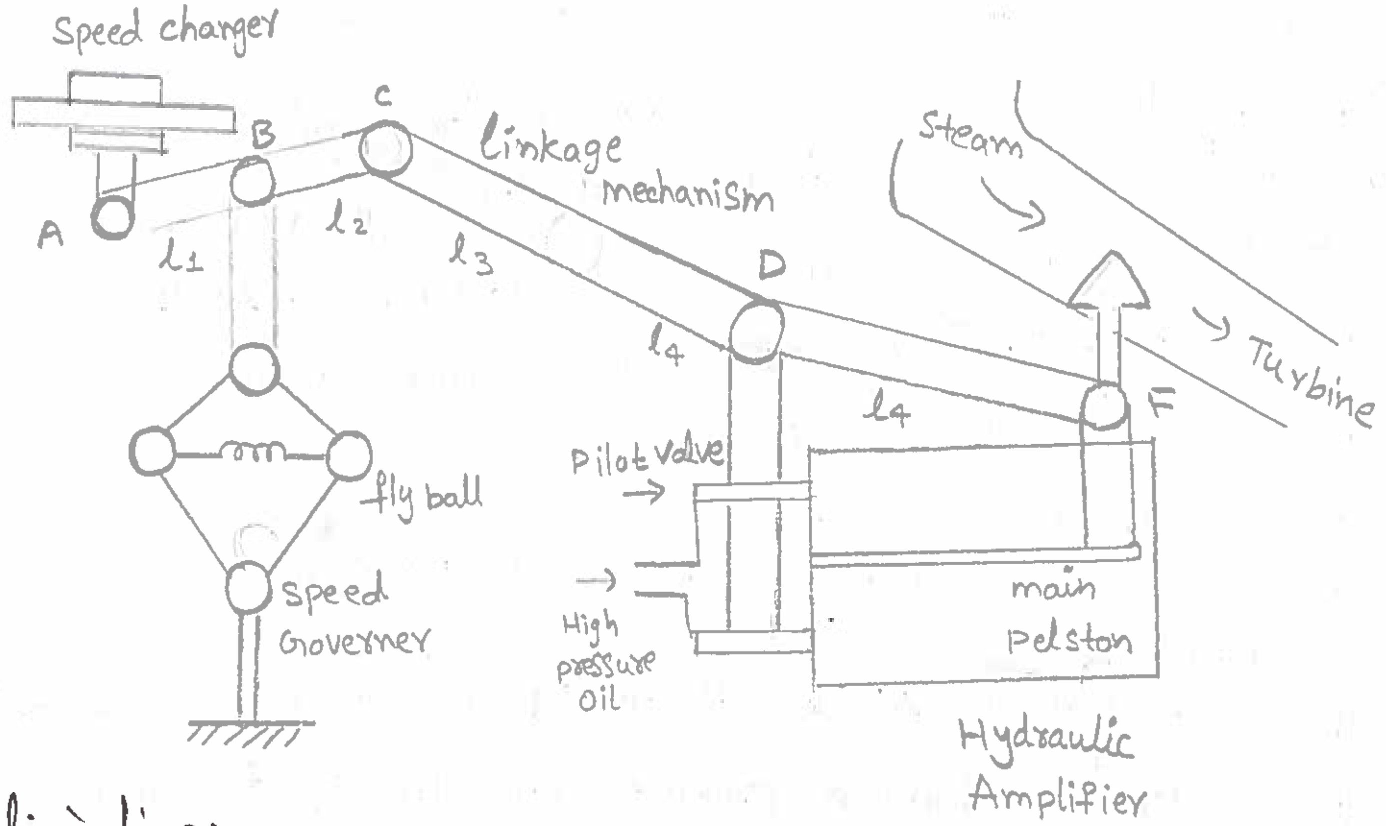
The dynamic interaction b/w pf control loop and Q-V control loop is due to the following two reasons.

1. change in bus-voltage at i^{th} bus changes the active power demand at i^{th} bus due to voltage load characteristics.

i.e $\dfrac{\partial P_{D,i}}{\partial |V_i|}$

2. The change in bus-voltage magnitude at i^{th} bus changes active power transmitting for the i^{th} transmission line.

Mathematical model of Speed Governor Systems:



Definiation:

Control Area: The generation is an Electrical area flows are Coherent group i.e all generators Speed up together are slow down together by maintaining their relatives load analysis, d

Analysis:

Consider an Speed governing System to Control active power flow in power System.

The speed governing system Consist Consist of 4 Components.

1. Speed governer
2. linkage mechanism
3. Hydralic amplifier
4. Speed changer.

1. Speed Governer:

, Speed governer senses the changes in speed (or) frequency

⇒ Therefore the speed governer is "Heart of the system".

, As the speed increses fly ball of speed governor move outwards and linkage point B move down downwards. As the speed decreses the fly ball of speed governer move inwards and linkage point B move upwards.

2. Linkage mechanism

> A. B C and C D E are the sid points are pionted at point B and point E.

, The lenth of suclions AB, BC, CD and DE are l_1, l_2 l_3 and l_4 respectivly

Rigid link mechanism provides movements to steam wall in propotional to change in speed.

The lenth of proulder feed back from steam value mument

3. Hydralic Amplifier

, Hydralic amplifier consist of main position and pilot walue

> lower power pilot value movement is converted into high power position valve which is neccessary to open (or) close the steam value against high pressure steam.

4. Speed changer:

The speed changes proulders steady state pore, output sching for the turbines.

> The downward movement of speed changes opens upper pilot value such that more steam emitted into turbine under steady condition.

⇒ The upwards movement of speed enchges closes upper pilot value so that less steam is adjusted adjusted into the turbine under steady state condition.

Modeling of Speed governer

Consider the steam is operating under steady state and is delivering power P_G from the generator at nominal frequency f.

Movement of point A

→ let us assume that raise Command ΔP_c to the speed changer the point A be moved downwards by a small amount Δx_A which causes the turbine power output to change.

$$\boxed{\therefore\ \Delta x_A = K_c\,\Delta P_c}$$

→ let us assume +ve direction downwards & -ve direction upwards.

Movement of point B:

→ An increse in frequency Δf causes the fly balls to move outwards. so that point B moves downwards by a propotional amount $K_2 \Delta f$ $\boxed{\Delta x_B = K_2\,\Delta f}$

Movement of point C:

→ An increse in frequency Δf causes the fly balls to move outward so that point B moves downwards by a propotional amount $K_2 \Delta f$

i) Δx_A Contributes $\left[-\frac{l_2}{l_1}\right] \Delta x_A = -K_1 K_c \Delta P_c$

ii) Δx_B Contributes $K_2 \Delta f$

$$\therefore\ \Delta x_c = -K_1 K_c\,\Delta P_c + K_2\,\Delta f \quad \text{——} (1)$$

Movement of point D:

- Its contributed by Δx_c and Δx_E the movement Δx_D is the amount by which the pilot valve opens, thereby moving the the main piston and opening the steam valve by Δx_E

$$\Delta x_D = \left[\frac{l_4}{l_3+l_4}\right]\Delta x_c + \left[\frac{l_3}{l_3+l_4}\right]\Delta x_E$$

$$\Delta x_D = k_3 \Delta x_c + k_4 \Delta x_E \quad \text{—— (2)}$$

movement of point E:

The value of oil admitted to the cylinder is thus proportional to the line integral of Δx_D

$$\Delta x_E = k_5 \int_0^1 (-\Delta x_D)\,dt \quad \text{—— (3)}$$

By taking laplace Transform of Equation (1), (2) & (3)

$$\Delta x_c(s) = -k_1 k_c \Delta P_c(s) + k_2 \Delta f(s) \quad \text{—— (4)}$$

$$\Delta x_D(s) = k_3 \Delta x_c(s) + k_4 \Delta x_E(s) \quad \text{—— (5)}$$

and $$\Delta x_E(s) = \frac{-k_5}{s}\Delta x_D(s) \quad \text{—— (6)}$$

Sub Eqn (5) in Eqn (6)

$$\Delta x_E(s) = \frac{-k_5}{s}\left[k_3 \Delta x_c(s) + k_4 \Delta x_E(s)\right]$$

$$\Delta x_E(s) = \frac{-k_5 k_3}{s}\Delta x_c(s) - \frac{k_4 k_5}{s}\Delta x_E(s)$$

$$\Delta x_E(s)\left[1 + \frac{k_4 k_5}{s}\right] = \frac{-k_5 k_3}{s}\Delta x_c(s) \quad \text{—— (7)}$$

Sub the Eqn 4 in Eqn 7

$$\boxed{\Delta X_C(s) = -K_1 K_C \Delta P_c + K_2 \Delta f(s)}$$

We get

$$\Delta X_E(s)\left[1+\frac{K_4 K_5}{s}\right] = \frac{-K_5 K_3}{s}\left[-K_1 K_C \Delta P_c + K_2 \Delta f(s)\right]$$

$$\Delta X_E(s)\left[\frac{s+K_4K_5}{s}\right] = \left[\frac{K_5 K_3 K_1 K_C \Delta P_c(s) - K_2 K_5 K_3 \Delta f(s)}{s}\right]$$

$$\Delta X_E(s)\left[s+K_4K_5\right] = \left[K_5 K_3 K_1 K_C \Delta P_c(s) - K_2 K_5 K_3 \Delta f(s)\right]$$

$$\Delta X_E(s)\, K_4K_5\left[1+\frac{s}{K_4K_5}\right] = K_5 K_3 K_1 K_C\left[\Delta P_c(s) - \frac{K_2}{K_1 K_C}\Delta f(s)\right]$$

$$\Delta X_E(s)\, K_4K_5\left[1+\frac{s}{K_4K_5}\right] = K_5 K_3 K_1 K_C\left[\Delta P_c(s) - \frac{K_2}{K_1 K_C}\Delta f(s)\right]$$

$$\Delta X_E(s) = \frac{K_5 K_3 K_1 K_C\left[\Delta P_c(s) - \frac{K_2}{K_1 K_C}\Delta f(s)\right]}{K_4 K_5\left[1+\frac{s}{K_4K_5}\right]}$$

$$\therefore\ \Delta X_E(s) = \frac{K_3 K_1 K_C}{K_4}\ \frac{\left[\Delta P_c(s) - \frac{K_2}{K_1 K_C}\Delta f(s)\right]}{1+\frac{s}{K_4 K_5}}$$

The above Eqn Can be written as

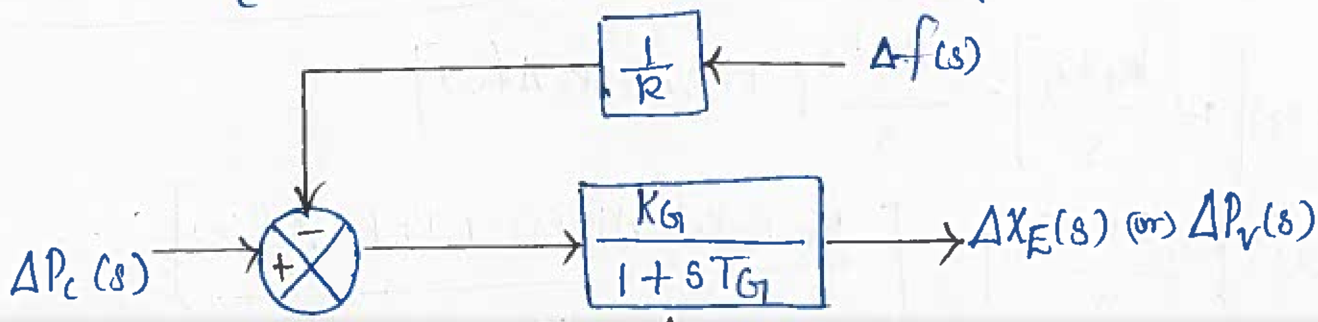
$$\Delta X_E(s) = \left[\Delta P_c(s) - \frac{1}{R}\Delta f(s)\right] \times \frac{K_G}{1+sT_G}$$

Where $R = \frac{K_1 K_C}{K_2}$ Speed regulation of governor Hz/MW

$K_G = \frac{K_1 K_3 K_C}{K_4}$ Gain of Speed governor

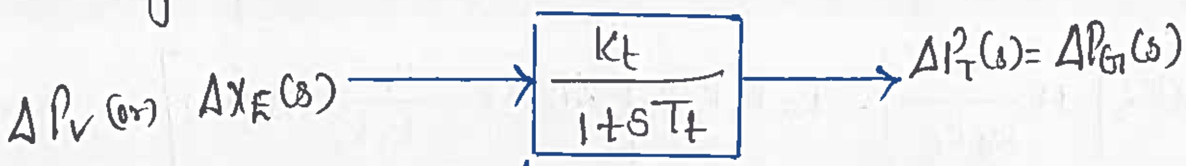
$T_G = \frac{1}{K_4 K_5}$ Time constant of Speed governor.

$$\Delta X_E(s) = \left[\Delta P_c(s) - \frac{1}{R}\Delta f(s) \right] \times \frac{K_G}{1+sT_G}$$



Model of speed governer

Modeling Turbine:

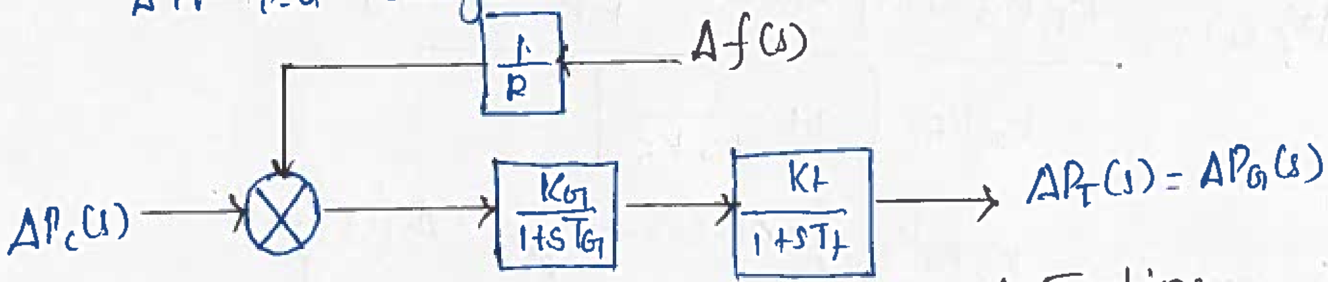


Model of Turbine

Where T_t = Time Constant of Turbine

K_t = Gain Constant

ΔP_v = P.U change in valve position from nominal value.



Combined model of speed governer and Turbine

Generator - load model:

⇒ Consider the mathematical modelof an isolation generator, which can only supplying local load and is not supplying power to another area.

⇒ Suppose there is a real load change of ΔP_D. Due to the action of the turbine Controllers, the generator increses in o/p by an amount ΔP_G.

⇒ The net surplus power $(\Delta P_G - \Delta P_D)$ will be absorbed by the system in two Catogories.

Case-1: By increasing the K.E in the rotor at the rate of $\frac{d}{dt}(W_{K.E})$ ⑧

$$W_{KE}^{\circ} = H \times P_r \cdot kW/sec \text{ --- } (9)$$

$$W_{KE}^{\circ} = \frac{I W_0^2}{2}$$

$$W_{KE}^{\circ} \propto f_0^2$$

$$W_{KE}^{\circ} \propto f_0^2 \text{ --- } (10)$$

$$W_{KE} \propto (f_0 + \Delta f)^2 \text{ --- } (11)$$

By dividing Eqn (11) by (10) We have

$$\frac{W_{KE}}{W_{KE}^{\circ}} = \left[\frac{f_0+\Delta f}{f_0}\right]^2 = W_{KE}^{\circ}\left[\frac{f_0+\Delta f}{f_0}\right] = W_{KE}^{\circ}\left[1+\frac{\Delta f}{f_0}\right]^2$$

$$W_{KE} = W_{KE}^{\circ}\left[1+\frac{\Delta f}{f_0}\right] = W_{KE}^{\circ}\left[1+\frac{2\Delta f}{f_0}+\frac{\Delta f^2}{f_0^2}\right]$$

$$W_{KE} = W_{KE}^{\circ}\left[1+\frac{2\Delta f}{f_0}\right]$$ [∵ Neglects Second order system]

Rate of change of KE

$$\frac{d\,W_{KE}}{dt} = \frac{2W_{KE}^{\circ}}{f_0}\frac{d}{dt}(\Delta f) \text{ --- } (12)$$

Sub Eqn (9) in (12)

$$\frac{d\,W_{KE}}{dt} = \frac{2H P_r}{f_0}\frac{d}{dt}(\Delta f) \text{ --- } (13)$$

Case-2: The rate of change of load w.r.to frequency

$$\frac{\partial P_D}{\partial f} = B \text{ --- } (14)$$

Where B = Damping Co-efficient in MW/Hz

value of damping Co-efficient is +ve value for monitoring load.

$$\boxed{\Delta P_G - \Delta P_D = B\,\Delta f} \text{ --- } (15)$$

By Consider the two Catogories the power balance Equation will be

$$\Delta P_G - \Delta P_D = \frac{2HP_r}{f_0} \frac{d}{dt}(\Delta f) + B\Delta f \quad \text{—— } 16$$

Now Deride with P_r

$$\Delta P_{G\,p.u} - \Delta P_{D\,p.u} = \frac{2H}{f_0} \frac{d}{dt}(\Delta f) + B_{p.u}\,\Delta f$$

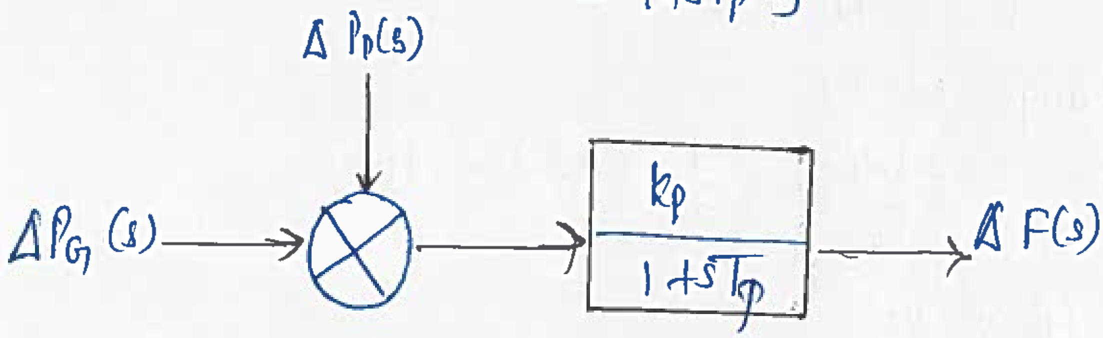
Taking laplace Transform on both sides we get.

$$\Delta P_G(s) - \Delta P_D(s) = \frac{2H(s)}{f_0} \Delta f(s) + B\Delta f(s)$$

$$\Delta P_G(s) - \Delta P_D(s) = \Delta f(s)\left[\frac{2HS}{f_0} + B\right]$$

$$\Delta P_G(s) - \Delta P_D(s) = \Delta f(s)\left[\frac{2HS}{f_0} + B\right]$$

$$\Delta f(s) = \Delta P_G(s) - \Delta P_D(s)\left[\frac{K_p}{1+sT_p}\right] \quad \text{—— } 17$$



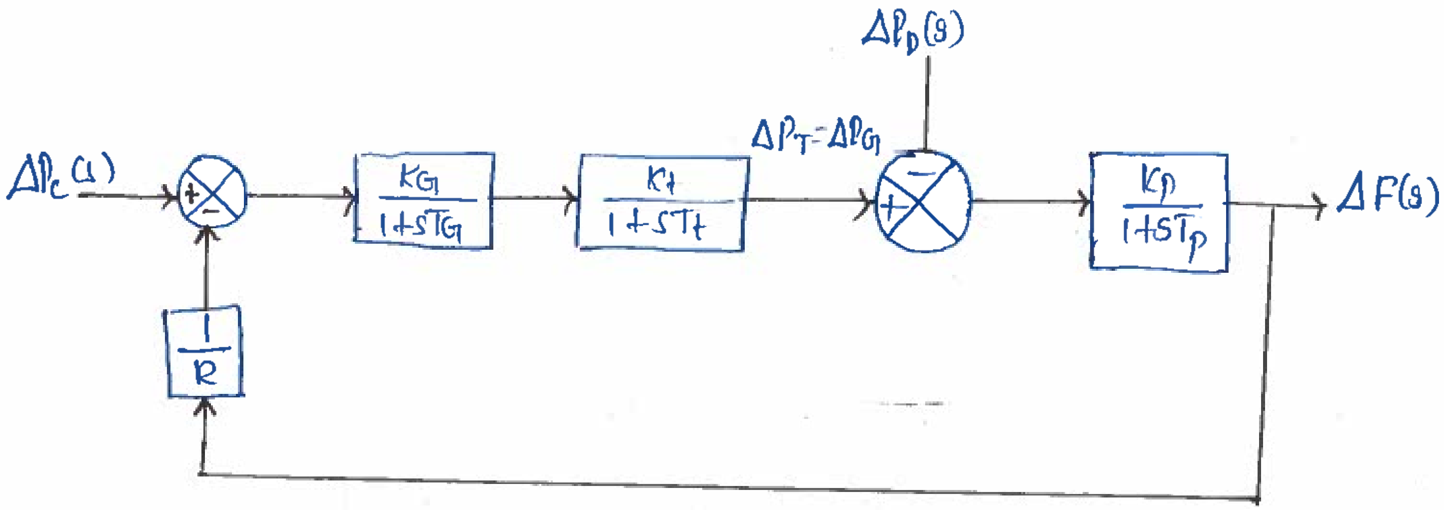
Where $K_p = \frac{1}{B}$ = Power System Gain

$T_p = \frac{2H}{Bf_0}$ = Power System time Constant.

Generator load model

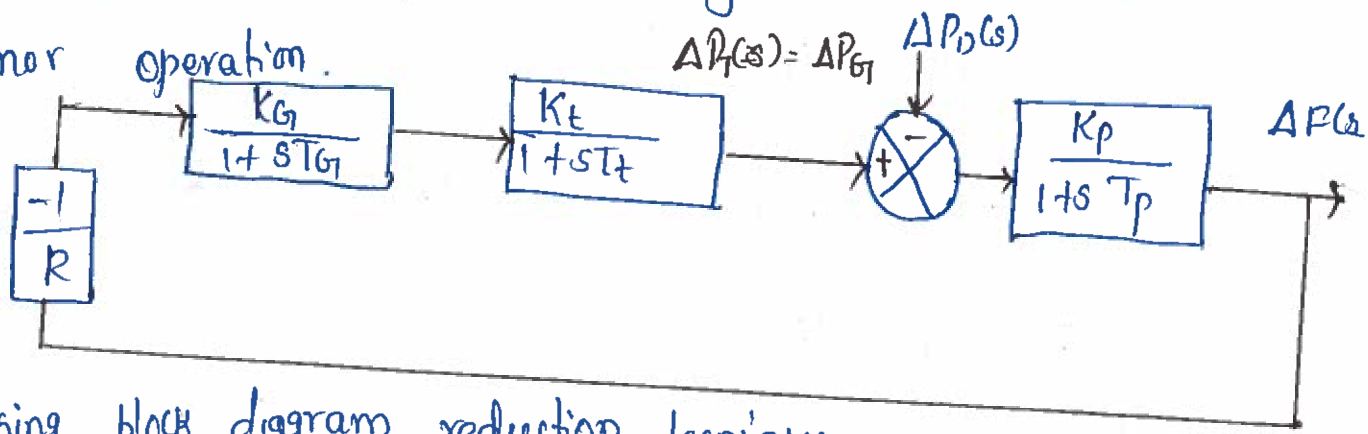
Model of load frequency Control of Single area System

The Combination of governer model, Turbine model and generator model is the Complete block diagram model of LFC of an Isolated Power System.

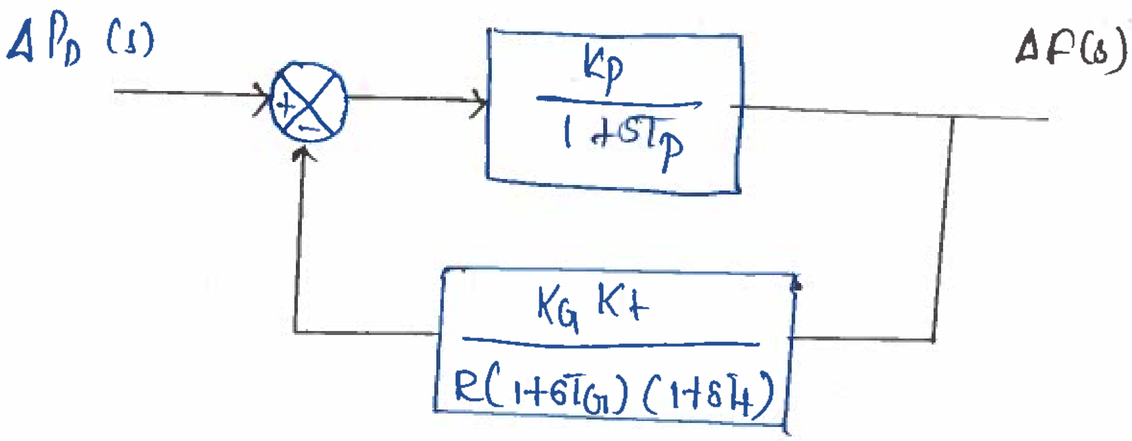


Case i: Uncontrolled Case $\Delta P_c = 0$

Consider the speed changes has a fixed setting. under this Condition $\Delta P_c = 0$ and the load demand changes this is known as free governor operation.



Using block diagram reduction technique



Transfer function $T.F = \dfrac{G(s)}{1+G(s)H(s)}$

$$\frac{\Delta F(s)}{-\Delta P_D(s)} = \frac{(K_P / 1+sT_P)}{1+\dfrac{K_P}{1+sT_P}\left[\dfrac{K_G K_t}{R(1+sT_G)(1+sT_t)}\right]}$$

$$\frac{\Delta F(s)}{-\Delta P_D(s)} = \frac{\left(\dfrac{K_P}{\cancel{1+sT_P}}\right)}{\dfrac{R(1+sT_P)(1+sT_G)(1+sT_t)+K_P K_G K_t}{R(\cancel{1+sT_P})(1+sT_G)(1+sT_t)}}$$

$$\frac{\Delta F(s)}{-\Delta P_D(s)} = \frac{K_P \cdot R(1+sT_G)(1+sT_t)}{R(1+sT_P)(1+sT_G)(1+sT_t)+K_P K_G K_t}$$

$$\Delta F(s) = \frac{-\Delta P_D(s) \cdot K_P \cdot R(1+sT_G)(1+sT_t)}{R(1+sT_P)(1+sT_G)(1+sT_t)+K_P K_G K_t}$$

for step input $\Delta P_D = \dfrac{\Delta P_D}{s}$

$$\Delta F(s) = \frac{-\dfrac{\Delta P_D}{s} \cdot K_P \cdot R(1+sT_G)(1+sT_t)}{R(1+sT_P)(1+sT_G)(1+sT_t)+K_P K_G K_t}$$

For static response, Apply final value theorem

$$\boxed{\Delta f_{stat} = \lim_{s\to 0} s \cdot \Delta F(s)}$$

$$\Delta f_{stat} = \lim_{s\to 0} \frac{\cancel{s} \cdot \dfrac{-\Delta P_D}{\cancel{s}} K_P \cdot R(1+sT_G)(1+sT_t)}{R(1+sT_P)(1+sT_G)(1+sT_t)+K_P K_G K_t}$$

$$\Delta f_{stat} = -\frac{\Delta P_D \cdot K_P \cdot R(1+0)(1+0)}{R(1+0)(1+0)(1+0)+K_P K_G K_t)}$$

$$\Delta f_{stat} = \frac{-\Delta P_D \cdot K_p \cdot R}{R + K_p K_G K_t}$$ [For static response practically $K_G K_t = 1$]

$$\Delta f_{stat} = \frac{-\Delta P_D \cdot K_p \cdot R}{R + K_p}$$

$$= \frac{-\Delta P_D \cdot \cancel{R} \cdot \left(\frac{1}{B}\right)}{\cancel{R}\left[1 + \frac{K_p}{R}\right]}$$

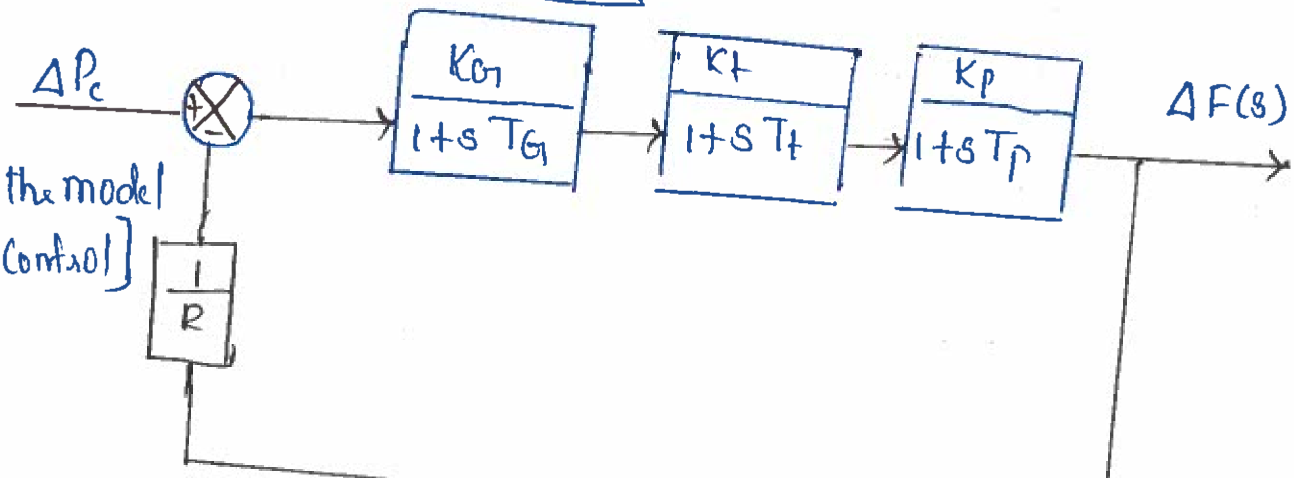
$$= \frac{-\Delta P_c \cdot \frac{1}{B}}{1 + \frac{1}{BR}}$$

$$\Delta f_{stat} = \frac{-\Delta P_D}{B + \frac{1}{R}}$$

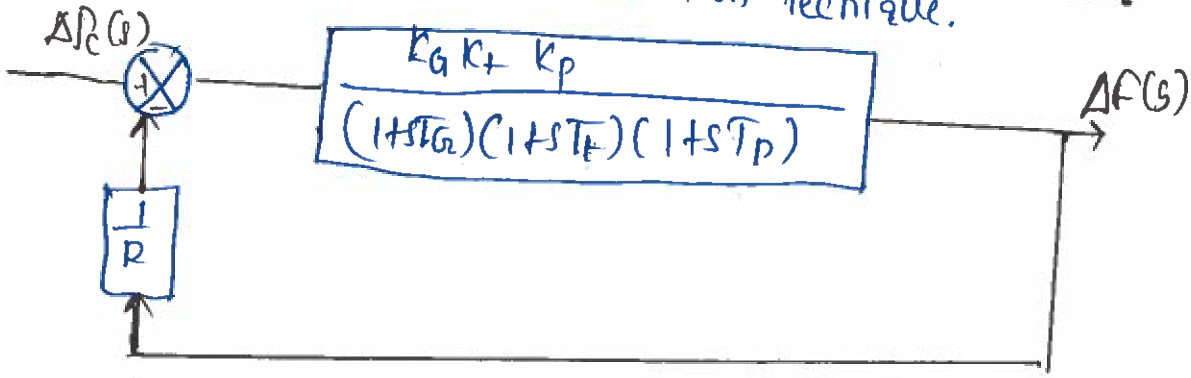
$$\boxed{\Delta f_{stat} = -R\,\Delta P_D} \qquad \boxed{\Delta P_c = 0}$$

Case ii Controlled Case $\boxed{\Delta P_D = 0}$

[∵ From the model of load-fr control]



using block diagram reduction Technique.



$$\text{Transfer function} = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{\Delta F(s)}{\Delta P_c(s)} = \frac{\dfrac{K_G K_t K_p}{(1+sT_G)(1+sT_t)(1+sT_p)}}{1+\dfrac{K_G K_t K_p}{R(1+sT_G)(1+sT_t)(1+sT_p)}}$$

$$\frac{\Delta F(s)}{\Delta P_c(s)} = \frac{\dfrac{K_G K_t K_p}{\cancel{(1+sT_G)}\cancel{(1+sT_t)}\cancel{(1+sT_p)}}}{\dfrac{R(1+sT_G)(1+sT_t)(1+sT_p)+K_G K_t K_p}{R\cancel{(1+sT_G)}\cancel{(1+sT_t)}\cancel{(1+sT_p)}}}$$

$$\frac{\Delta F(s)}{\Delta P_c(s)} = \frac{R \cdot K_G K_t K_p}{R(1+sT_G)(1+sT_t)(1+sT_p)+K_G K_t K_p}$$

$$\Delta F(s) = \frac{\Delta P_c(s) \cdot R \cdot K_G K_t K_p}{R(1+sT_G)(1+sT_t)(1+sT_p)+K_G K_t K_p}$$

$$\Delta F(s) = \frac{\Delta P_c(s) \cdot R \cdot K_G K_t K_p}{R(1+sT_G)(1+sT_t)(1+sT_p)+K_G K_t K_p}$$

for step input $\Delta P_c(s) = \dfrac{\Delta P_c}{s}$

$$\Delta F(s) = \frac{\dfrac{\Delta P_c}{s} \cdot R \cdot K_G K_t K_p}{R(1+sT_G)(1+sT_t)(1+sT_p)+K_G K_t K_p}$$

For static Apply final value theorem

$$\Delta f_{stat} = \lim_{s \to 0} s . \Delta F(s)$$

$$\Delta f_{stat} = \frac{\lim_{s \to 0} s . \frac{\Delta P_c}{s} \quad R . K_G K_t K_p}{R(1+sT_G)(1+sT_t)(1+sT_p) + K_G K_t K_p}$$

$$\Delta f_{stat} = \frac{\Delta P_c . R . K_G K_t K_p}{R + K_G K_t K_p} \quad [\because K_G K_t = 1]$$

$$\Delta f_{stat} = \frac{\Delta P_c . R . K_p}{R + K_p} = \frac{\Delta P_c . R . K_p}{R\left[1 + \frac{K_p}{R}\right]} = \frac{\Delta P_c . \frac{1}{B}}{1 + \frac{1}{BR}}$$

$$\Delta f_{stat} = \frac{\Delta P_c}{B + \frac{1}{R}} \quad [\because B << \tfrac{1}{R} ; \text{ neglect } B]$$

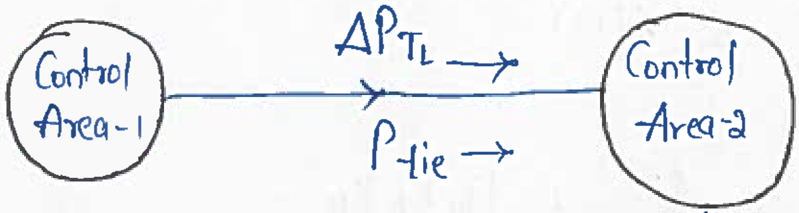
$$= \frac{\Delta P_c}{\frac{BR+1}{R}}$$

$$= \frac{R . \Delta P_c}{BR+1}$$

$$\Delta f_{stat} = R . \Delta P_c$$

$$\boxed{\Delta f_{stat} = R . \Delta P_c} \quad [\Delta P_D = 0]$$

Two-Area load frequency Control



An Extended Power System Can be divided into a number of load frequency Control areas by means of Transmission lines (or) Tie-lines. Such as operation is known as power pool operation.

Objective:

The objective is to regulate the frequency of each area and to Simultaneously regulate the power flow through Transmission line.

Single Control Area

→ The integral Controller gives zero steady state Error in frequency

→ The Incremental Power:

$$\Delta P_G - \Delta P_D = \frac{dW_{KE}}{dt} + B\Delta f$$

Two Control Area.

→ Integral Controller gives zero Steady state Error in frequency and transmission line power flow.

→ The Incremental Power

$$\Delta P_G - \Delta P_D = \frac{dW_{KE}}{dt} + B\Delta f + \Delta P_{TL}$$

Let $|E_1|\angle\delta_1$, $|E_2|\angle\delta_2$ be the Excitation emf magnitudes and their phase angles related to Control areas 1 & 2 respectively

Power flow out of Control area-1.

$$P_{TL\,1} = \frac{|E_1|\,|E_2|}{X_{TL}} \sin(\delta_1 - \delta_2)$$

As the load demand changes in a Control area, the load angle of Control areas 1 & 2 changes.

→ let $\Delta\delta_1$ and $\Delta\delta_2$ be the incremental change in a Control Area the load angle of Control areas 1 & 2 changes.

→ Let $\Delta\delta_1$ and $\Delta\delta_2$ be the increme

The total power flow out of Control Area-1

$$P_{TL1} + \Delta P_{TL,1} = \frac{|E_1||E_2|}{X_{TL}} \sin\left[(\delta_1-\delta_2) + (\Delta\delta_1 - \Delta\delta_2)\right]$$

$$= \frac{|E_1||E_2|}{X_{TL}} \left[\begin{array}{l} \sin(\delta_1-\delta_2)\cos(\Delta\delta_1-\Delta\delta_2) + \\ \cos(\delta_1-\delta_2)\sin(\Delta\delta_1-\Delta\delta_2) \end{array}\right]$$

As the changes in the load angle are small

$$\Delta\delta_1 - \Delta\delta_2 \cong 0$$

$$\cos(\Delta\delta_1 - \Delta\delta_2) = \cos 0 = 1$$

$$\sin(\Delta\delta_1 - \Delta\delta_2) = \Delta\delta_1 - \Delta\delta_2$$

$$P_{TL1} + P_{TL,1} = \frac{|E_1||E_2|}{X_{TL}}\sin(\delta_1-\delta_2) + \frac{|E_1||E_2|}{X_{TL}}\cos(\delta_1-\delta_2)(\Delta\delta_1-\Delta\delta_2) \quad -(1)$$

Comparing RHS and LHS.

Incremental change in tie-line Power out of Area-1

Devide LHS and RHS by rated Power of Control Area-1

$$\Delta P_{TL,1} = \frac{|E_1||E_2|}{X_{TL}} \cos(\delta_1-\delta_2)(\Delta\delta_1-\Delta\delta_2)$$

$$\frac{\Delta P_{TL1}}{P_{rated,1}} = \frac{|E_1||E_2|}{X_{TL} \cdot P_{rated}} \cos(\delta_1-\delta_2)(\Delta\delta_1-\Delta\delta_2)$$

$$\boxed{\frac{\Delta P_{TL,1}}{P_{rated,1}} = T_{12}(\Delta\delta_1-\Delta\delta_2)}$$

$$\boxed{T_{12} = \frac{|E_1||E_2|}{X_{TL}\ P_{rated,1}} \cos(\delta_1-\delta_2)}$$

T_{12} is Synchronising Power Co-efficient of the Transmission line consider, the change in frequency

$$\Delta W = \frac{d}{dt}(\Delta\delta)$$

$$2\pi \cdot \Delta f = \frac{d}{dt}(\Delta\delta)$$

$$\Delta f = \frac{1}{2\pi}\frac{d}{dt}(\Delta\delta)$$

On integrating, $\Delta\delta = 2\pi \int \Delta f \, dt$ radians

Accordingly the incremental change in load angle of area-1 and area-2 are

$$\boxed{\begin{aligned}\Delta\delta_1 &= 2\pi \int \Delta f_1 \, dt \\ \Delta\delta_2 &= 2\pi \int \Delta f_2 \, dt\end{aligned}} \quad \text{—— (4)}$$

Substitute Eqn (4) in Eqn (2)

$$\boxed{\Delta P_{TL,1} \text{ P.U} = 2\pi \cdot T_{12}\left[\int \Delta f_1 dt - \int \Delta f_2 dt\right]}$$

Where Δf_1 and Δf_2 are the incremental changes in frequency of control areas-1 and 2 respectively

The increment changes in transmission line power flowing out of control area-2

$$\boxed{\Delta P_{TL,2} \text{ P.U} = 2\pi\, T_{21}\left[\int \Delta f_2 dt - \int \Delta f_1 dt\right]}$$

Where $\boxed{T_{21} = \frac{|E_2||E_1|}{X_{TL}, P_{rated}} \cos(\delta_2 - \delta_1)}$

$$\frac{(3)}{(7)} : \frac{T_{12}}{T_{21}} = \frac{P_{rated,2}}{P_{rated\,1}} = \frac{1}{a_{12}} \text{ (say)}$$

$$\boxed{\begin{aligned} T_{21} &= a_{12} T_{12} \\ \Delta P_{TL,2} &= a_{12} \cdot \Delta P_{TL,2} \end{aligned}} \quad \text{--- (8)}$$

For two control area, the surplus power can be expressed in per unit as

$$\boxed{\Delta P_{G_1 \, p.u} - \Delta P_{G_2 \, p.u} = \frac{2H}{f} \frac{d}{dt} (\Delta f_1) + B_1 \Delta f_1 + \Delta P_{TL,1} \, p.u} \quad \text{--- (9)}$$

Applying laplace Transforms

$$\Delta P_{G_1}(s) - \Delta P_{D_1}(s) = \frac{2H}{f^{\circ}} (s \, \Delta F_1(s)) + B_1 \Delta F_1(s) + \Delta P_{TL,1}(s)$$

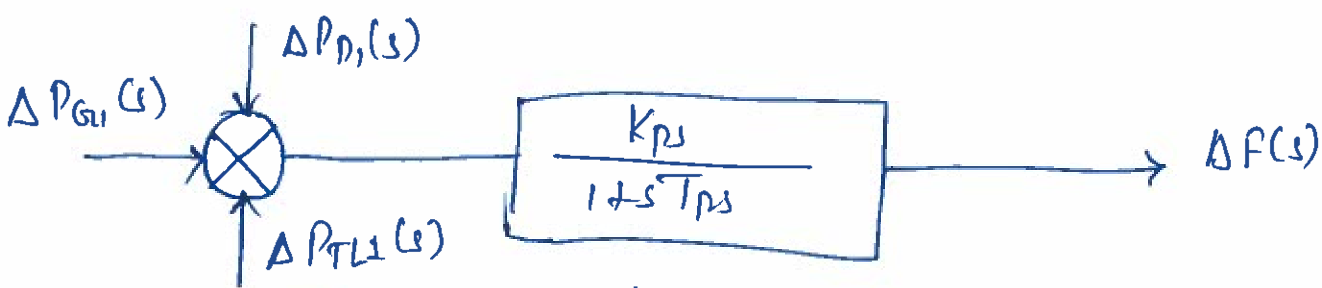
$$\Delta F_1(s) \left[\frac{2Hs}{f^{\circ}} + B_1 \right] = \Delta P_{G_1}(s) - \Delta P_D(s) - \Delta P_{TL1}(s)$$

$$\Delta F_1(s) \cdot B \left[1 + \frac{2H}{B_1 f} \cdot s \right] = \Delta P_{G_1}(s) - \Delta P_{D_1}(s) - \Delta P_{TL1}(s)$$

$$\Delta F_1(s) = \left[\frac{\frac{1}{B}}{\left(1 + \frac{2H}{B_1 f^{\circ}} s\right)} \right] (\Delta P_{G_1}(s) - \Delta P_{D_1}(s) - \Delta P_{TL_1}(s))$$

$$\boxed{\Delta F_1(s) = \left(\frac{K_{ps}}{1 + s T_{ps}} \right) (\Delta P_{G_1}(s) - \Delta P_{D_1}(s) - \Delta P_{TL1}(s))} \quad \text{--- (10)}$$

Eqn(10) can be represented using block diagram as follows



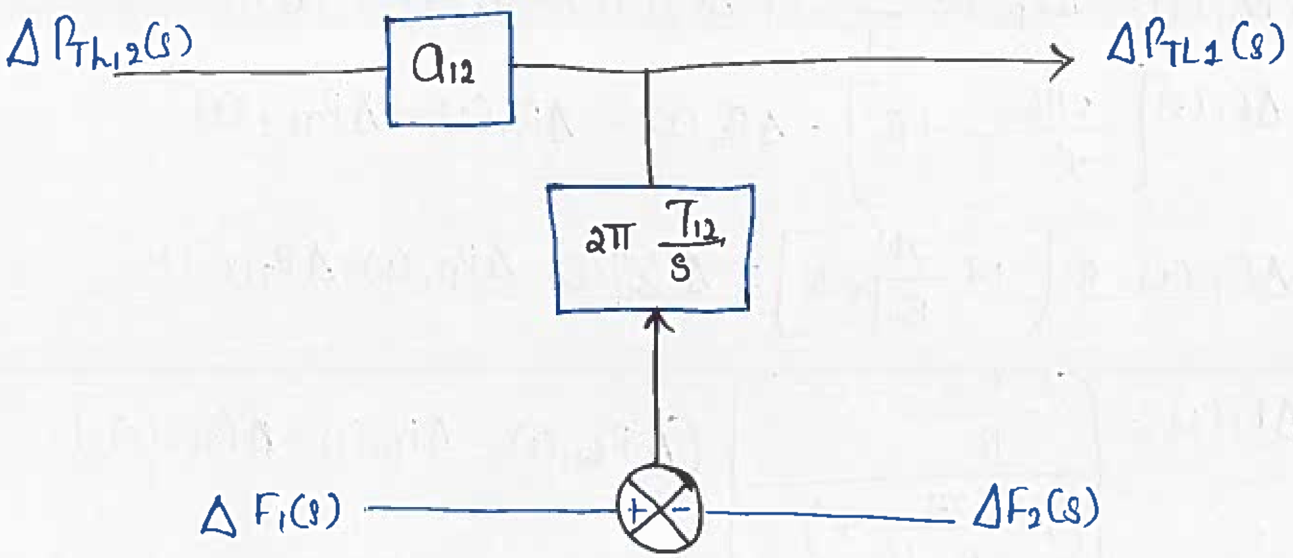
Applying laplace Transform to Eqn (5)

$$\boxed{\Delta P_{TL\,1}(s) = \frac{2\pi T_{12}}{s} \left[\Delta F_1(s) - \Delta F_2(s) \right]} \quad \text{--- (11)}$$

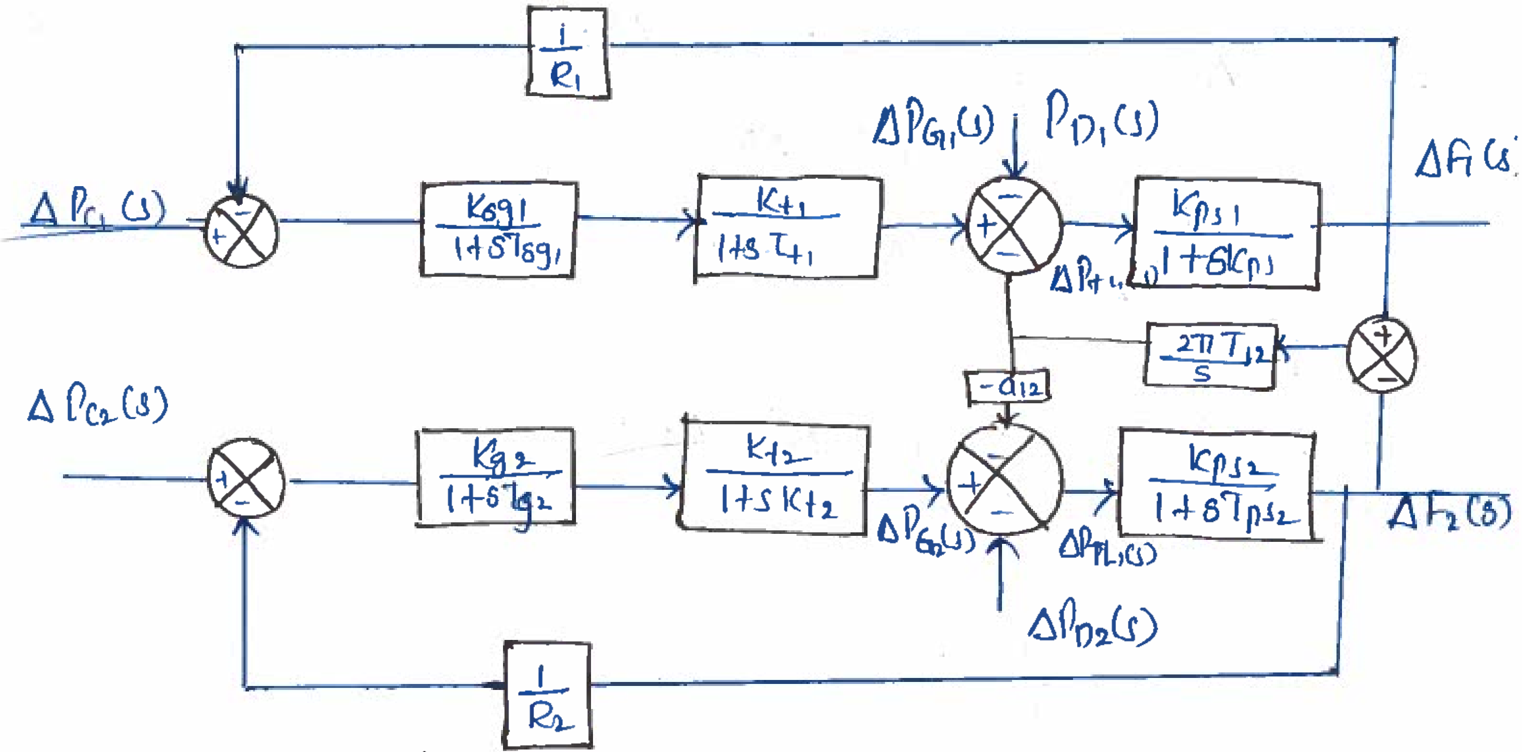
Applying laplace Transform to Eqn 6

$$\boxed{\begin{aligned}\Delta P_{TL,2}(s) &= \frac{2\pi T_{21}}{s}\left(\Delta f_2(s) - \Delta f_1(s)\right) \\ &= \frac{-2\pi a_{12} T_{12}}{s}\left[\Delta f_1(s) - \Delta f_2(s)\right]\end{aligned}} \quad \text{— (12)}$$

The block diagram for the Equation (11) & (12) are as follows



P-F Control of Two-Area System



Case i: Uncontrolled Case

The speed changer position are fixed i.e $\Delta P_{c_1}(s)=0$ and

$\Delta P_{c_2}(s) = 0$

static response (or) steady state response

$\Delta f_1 = \Delta f_2 = \Delta f$ (say)

As the static change in the frequency are to be determined the incremental changes in generation.

$$\Delta P_{G_1} = \frac{-\Delta f}{R_1} \quad \text{---} (1)$$

$$\Delta P_{G_2} = \frac{-\Delta f}{R_2} \quad \text{---} (2)$$

under steady state condition

$$\frac{d}{dt}(\Delta f_1) = \frac{d}{dt}(\Delta f_2) = 0 \quad \text{---} (3)$$

for 2-Area system the dynamics are described by

$$\Delta P_{G_1} - \Delta P_{D_1} = \frac{2H_1}{f^0}\frac{d}{dt}(\Delta f_1) + B_1 \Delta f_1 + \Delta P_{T_1} \quad \text{---} (4)$$

$$\Delta P_{G_2} - \Delta P_{D_2} = \frac{2H_2}{f_0}\frac{d}{dt}(\Delta f_2) + B_2 \cdot \Delta f_2 + \Delta P_{TL2} \quad \text{---} (5)$$

Sub Eqn (1,2,3,4) in (4,5)

$$-\frac{\Delta f}{R_1} - \Delta P_{D_1} = \frac{2H}{f_0}(0) + B_1 \Delta f + \Delta P_{TL_1}$$

$$-\frac{\Delta f}{R_1} - \Delta P_{D_1} = B_1 \Delta f + \Delta P_{TL_1} \quad \text{— (6)}$$

$$-\frac{\Delta f}{R_1} - \Delta P_{D_2} = B_2 \Delta f + \Delta P_{TL2} \quad \text{— (7)}$$

$$-\frac{\Delta f}{R_2}$$

from Eqn (6)

$$\Delta P_{LT_1} = -\left(\frac{1}{R_1} + B_1\right)\Delta f - P_{D_1} \quad \text{— (8)}$$

from Eqn (7)

$$-\frac{\Delta f}{R_2} = -\left(\frac{1}{R_1} + B_1\right.$$

$$-\frac{\Delta f}{R_2} - \Delta P_{D_2} = B_2 \Delta f - a_{12} \Delta P_{L1}$$

Sub Eqn (8)

$$-\frac{\Delta f}{R_2} - \Delta P_{D_2} = B_2 \Delta f - a_{12}\left[-\left(\frac{1}{R_1} + B_1\right)\Delta f - \Delta P_{D_1}\right]$$

$$-\frac{\Delta f}{R_2} - \Delta P_{D_2} = B_2 \Delta f + a_{12}\left(\frac{1}{R_1} \; B_1\right)\Delta f + a_{12} \Delta P_{D_1}$$

$$\Delta f\left[\left(B_2 + \frac{1}{R_2}\right) + a_{12}\left(\frac{1}{R_1} + B_1\right)\right] = -\left(\Delta P_{D_2} + a_{12} \Delta P_{D_1}\right)$$

$$\boxed{\Delta f = \frac{-\left(\Delta P_{D_2} + a_{12} \Delta P_{D_1}\right)}{\left(B_2 + \frac{1}{R_2}\right) + a_{12}\left(B_1 + \frac{1}{R_1}\right)} = \frac{-\left(\Delta P_{D_2} + a_{12} \Delta P_{D_1}\right)}{B_2 + a_{12} \beta_1}} \quad (9)$$

Sub Eqn (9) in Eq (8)

$$\Delta P_{TL_1} = -\left(\frac{1}{R_1} + B_1\right)\Delta f - \Delta P_{D_1}$$

$$\Delta P_{TL_1} = -\left(\frac{1}{R_1} + B_1\right)\Delta f - \Delta P_{D_1}$$

$$= -\left(\frac{1}{R_1} + B_1\right)\left[\frac{-(\Delta P_{D_2} + a_{12}\Delta P_{D_1})}{\left(\frac{1}{R_2} + B_2\right) + a_{12}\left(B_1 + \frac{1}{R_1}\right)}\right] - \Delta P_{D_1}$$

$$= \frac{\left(B_1 + \frac{1}{R_1}\right)\Delta P_{D_2} + \left[B_1 + \frac{1}{R_1}\right]a_{12}\Delta P_{D_1} - \left(B_2 + \frac{1}{R_2}\right)\Delta P_{D_1} - a_{12}\left(B_1 + \frac{1}{R_1}\right)\Delta P_{D_1}}{\left(B_2 + \frac{1}{R_2}\right) + a_{12}\left(B_1 + \frac{1}{R_1}\right)}$$

$$= \frac{\left(B_1 + \frac{1}{R_1}\right)\Delta P_{D_2} - \left(B_2 + \frac{1}{R_2}\right)\Delta P_{D_1}}{\left(B_2 + \frac{1}{R_2}\right) + a_{12}\left(B_1 + \frac{1}{R_1}\right)}$$

$$= \frac{\left(B_1 + \frac{1}{R_1}\right)\Delta P_2 - \left(B_2 + \frac{1}{R_2}\right)\Delta P_{D_1}}{\left(B_2 + \frac{1}{R_2}\right) + a_{12}\left(B_1 + \frac{1}{R_1}\right)}$$

$$\boxed{\Delta P_{TL_1} = \frac{\beta_1 \Delta P_{D_2} - \beta_2 \Delta P_1}{\beta_2 + a_{12}\beta_1}} \quad \text{— } 10$$

Equation 9, 10 gives the static changes in the frequency and tie-line power as a result of sudden step load change in the two areas.

Condition-1:

let Control area-1 and Control area-2 be identical

$B_1 = B_2 = B \qquad R_1 = R_2 = B \qquad \beta_1 = \beta_2 = \beta \qquad a_{12} = 1$

9: $\Delta f = \dfrac{-(\Delta P_{D_2} + \Delta P_{D_1})}{\beta + \beta} = -\dfrac{(\Delta P_{D_2} + \Delta P_{D_1})}{2\beta}$ — 11

10: $\Delta P_{TL_1} = \dfrac{\beta\Delta P_{D_2} - \beta\Delta P_{D_1}}{\beta + \beta} = \dfrac{\Delta P_{D_2} - \Delta P_{D_1}}{2}$ — 12

$\Delta P_{TL_2} = -a_{12}\,\Delta P_{TL_1 p.u} = -\Delta P_{TL_1 p.u}$ — 13

Condition-2:

Assume that sudden change in load demand occurs only in Control area-2.

$\Delta P_{D_1} = 0$

10: $\Delta f = \dfrac{-\Delta P_{D_2}}{\beta+\beta} = \dfrac{-\Delta P_{D_2}}{2\beta}$ —— (14)

11: $\Delta P_{L_1} = \dfrac{\beta \; \Delta P_{D_2}}{\beta+\beta} = \dfrac{\Delta P_{D_2}}{2}$ —— (15)

Advantages of Power pool operation:

1. The change in frequency Δf for two area system $= \frac{1}{2} \times$ change in frequency with one controlled area.
2. With two area load frequency Control, half of the load in area-2 is supplied by area-1 through the tie-line.

problem:

The load flow data for the Sample System are given below the voltage magnitude at bus-2 is to be mainted at 1.04 p.u the max and min reactive power limits of the generated at bus-2 are 0.35 and 0.0 p.u respectivly. Determine the set of load flow Equation at the end of the first iteration by Newton Rapson method.

Impedence for Sample System

Bus Code	Impedence	line charging admitance
1-2	0.08 + j0.24	0.0
1-3	0.02 + j0.06	0.0
2-3	0.06 + j0.18	0.0

Shedule of generation of loads

Bus Code	Assume voltage	Generation MW	Generation MVAR	load MW	load MVR.
1	1.06 + j0.0	0	0	0	0
2	1.0 + j0.0	0.2	0	0	0
3	1.0 + j0.0	0	0	0.6	0.25

sol

$$Y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.08 + j0.24} \times \frac{0.08 - j0.24}{0.08 - j0.24}$$

$$Y_{12} = 1.25 - j3.75$$

$$Y_{31} = Y_{13} = 5 - j15 \quad \& \quad Y_{23} = Y_{32} = 1.667 - j5.0$$

$$Y_{11} = Y_{12} + Y_{13} = 6.25 - j18.75$$

$$Y_{22} = Y_{21} + Y_{23} = 2.916 - j8.75$$

$$Y_{33} = Y_{31} + Y_{32} = 6.666 - j20$$

then the Y_{bus} Matrix is

$$Y_{bus} = \begin{bmatrix} 6.25-j18.75 & -1.25+3.75 & -5+j15 \\ -1.25+j3.75 & 2.916-j8.75 & -1.666+j50 \\ -5+j15 & -1.666+j50 & 6.666+j20 \end{bmatrix}$$

Assuming a flat voltage profile for bus 2 and bus-3 and for bus-1

$V_1 = 1.06 + j0.0$

For the nodal admitance matrix and assuming voltage solution

$$\boxed{Y = G - jB}$$

$G_{11} = 6.25$; $B_{11} = 18.75$

$G_{12} = -1.25$; $B_{12} = -3.75$

$G_{13} = -5.0$; $B_{13} = -15.0$

$G_{22} = 2.916$; $B_{22} = 8.75$

$G_{23} = -1.666$; $B_{23} = -5.0$

$G_{33} = 6.666$; $B_{33} = 20$

$e_1 = 1.06$ $\quad f_1 = 0.0$

$e_2 = 1.0$ $\quad f_2 = 0.0$

$e_3 = 1.0$ $\quad f_3 = 0.0$

$$P_p = \sum_{q=1}^{n} \left[e_p (e_q G_{pq} + f_q B_{pq}] + f_p (f_q G_{pq} - e_q B_{pq}) \right.$$

$$p = 2 ; \quad q = 1, 2, 3$$

$$P_2 = [e_2 (e_1 G_{21} + f_1 B_{21}) + f_2 (f_1 G_{21} - e_1 B_{21})]$$
$$+ e_2 (e_2 G_{22} + f_2 B_{22}) + f_2 (f_2 G_{22} - e_2 B_{22})$$
$$+ e_2 (e_2 G_{23} + f_2 B_{23}) + f_2 (f_3 G_{23} - e_3 B_{23})]$$

$P_2 = -0.075$ P.U

Similarly

$P_3 = -0.3$

$$Q_p = \sum_{q=1}^{n} [f_p (e_q G_{pq} + f_q B_{pq}) - e_p (f_q G_{pq} - e_q B_{pq})]$$

$$Q_2 = [f_2 (\qquad) - e_2 (f_1 G_{21} - e_1 B_{21}) + f_2 (0)$$
$$- e_2 (f_2 G_{22} - e_2 B_{22}) + f_2 (0)$$
$$- e_2 (f_3 G_{23} - e_3 B_{23})$$

$Q_2 = -0.225$ P.U

Similarly

$Q_3 = -0.9$

$$\Delta P_2 = P_2\,sp - P_2\,cal$$
$$= 0.2 - (-0.075$$
$$= 0.275 \text{ P.U}$$

$$\Delta P_3 = -0.6 - (-0.3)$$
$$= -0.3 \text{ P.U}$$