Discrete Mathematics

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UNIT-V

GRAPH THEORY

CONTENT..

Graph Theory: Basic Concepts
Isomorphism and Subgraphs
Trees and their Properties
Spanning Trees
Directed Trees, Binary Trees
Planar Graphs
Euler's Formula
Multi-graphs and Euler Circuits
Hamiltonian Graphs
Chromatic Numbers
The Four-Color Problem.

Basic Concepts of Graphs

Applications:
Computer networks (nodes & links)
Power grid systems
Social networks
Transportation networks
Model Problem:
Problem:
A network has 6 computers connected by 7 communication links. Represent this system as a graph and find the number of vertices and edges.
Solution:
Vertices = 6
Edges = 7
✓ **Answer:** Graph has 6 vertices and 7 edges.

Isomorphism and Subgraphs

Applications:
Pattern recognition
Circuit comparison
Chemical structure analysis
Software architecture matching
Model Problem:
Problem:
Determine whether the two graphs below with same degree sequence ⟨ 3,3,2,2⟩ are isomorphic.
Solution:
Same degree sequence
One-to-one correspondence between vertices preserving adjacency
✓ **Answer:** The graphs are **isomorphic**.

Planar Graphs

Applications:
VLSI circuit design
PCB layout
City planning
Network visualization
Model Problem:
Problem:
Check whether a graph with 5 vertices and 9 edges can be planar.
Solution:
Planar graph condition:

$$e \leq 3v - 6$$
$$9 \leq 3(5) - 6 = 9$$

✓ **Answer:** The graph **can be planar**.

Euler's Formula

Applications:
Surface modeling
Computer graphics
Polyhedron design
Mesh generation
Model Problem:
Problem:
For a planar graph with 10 vertices and 16 edges, find the number of faces.
Solution:
Euler's formula:

$$v - e + f = 2$$
$$10 - 16 + f = 2 \Rightarrow f = 8$$

✓ **Answer:** 8 faces

Multigraphs and Euler Circuits

Applications:
Road networks
Communication links
Power grids
Logistics networks
Model Problem:
Problem:
State the condition for a graph to have an Euler circuit.
Solution:
A graph has an Euler circuit if:
It is connected
All vertices have even degree
✓ **Answer:** Euler circuit exists if all vertices have even degree.

Hamiltonian Graphs

Applications:
Traveling salesman problem
Robotics path planning
DNA sequencing
Network routing
Model Problem:
Problem:
Does a complete graph K_5 have a Hamiltonian circuit?
Solution:
Every complete graph with $n \geq 3$ is Hamiltonian.
✓ **Answer:** Yes, K_5 is Hamiltonian.

rees and Their Properties

Applications:
File systems
Organizational hierarchy
Decision trees
XML/HTML parsing
Model Problem:
Problem:
Find the number of edges in a tree with 10 vertices.
Solution:
Property of tree:

$$\text{Edges} = n - 1$$
$$= 10 - 1 = 9$$

✓ **Answer:** 9 edges

Spanning Trees

Applications:
Network design
Cost-effective wiring
Power distribution
Minimum connection systems
Model Problem:
Problem:
How many edges are there in a spanning tree of a graph with 8 vertices?
Solution:
A spanning tree is a tree with all vertices:

$$\text{Edges} = n - 1 = 7$$

✓ **Answer:** 7 edges

Directed Trees

Applications:
Control flow graphs
Compiler design
Task scheduling
Dependency management
Model Problem:
Problem:
Define a directed tree and state how many edges it has if it contains 12 vertices.
Solution:
A directed tree has:

$$\text{Edges} = n - 1 = 11$$

✓ **Answer:** 11 edges

Binary Trees

Applications:
Searching algorithms
Expression evaluation
Databases (B-trees)
AI decision making
Model Problem:
Problem:
Find the maximum number of nodes in a binary tree of height 4.
Solution:

$$\text{Max } nodes = 2^{h+1} - 1$$

$$= 2^5 - 1 = 31$$

✓ **Answer:** 31 nodes

Chromatic Number

Applications:

Register allocation

Frequency assignment

Timetable scheduling

Map coloring

Model Problem:

Problem:

Find the chromatic number of a complete graph K_4.

Solution:

Chromatic number of $K_n = n$

✓ **Answer:** 4

Four-Color Problem

Applications:
Map coloring
GIS systems
Resource allocation
Conflict-free scheduling
Model Problem:
Problem:
State the Four-Color Theorem.
Solution:
Any planar map can be colored using **at most four colors** such that no two adjacent regions have the same color.
✓ **Answer:** Four colors are sufficient.

BFS (Breadth First Search)

Problem:

Given the graph with vertices

$$V = \{A, B, C, D, E, F\}$$

and edges

$\{A, B, A, C, B, DBECFEF\}$.

Perform **BFS traversal starting from vertex A**.

Solution:

Step 1: Start at A

Visit A

Queue: A

Step 2: Visit neighbors of A

Neighbors: B, C

Queue: B, C

Step 3: Visit B

DFS (Depth First Search)

Problem:
Using the same graph, perform **DFS traversal starting from vertex A**.

Solution:
Step 1: Start at A
Step 2: Go deep before backtracking
Traversal:
A → B → D (backtrack)
B → E (backtrack)
A → C → F

✓ **DFS Traversal Order:**

THANK YOU..