UNIT-2

Set Theory

Discrete Mathematics

Subject Code:25CS301

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TOPICS TO BE COVERED

Set theory:
Introduction
Basic Concepts of Set Theory
Representation of Discrete Structures
Relations and Ordering, Functions.

Basic Concepts of Set Theory

2.1 Definition of a Set

Definition of a Set

A **set** is a well-defined collection of distinct objects, called **elements** or **members**.

Example:

A = {1, 2, 3}

B = {a, e, i, o, u}

Notation:

If a is an element of set A, then $a \in A$

If not, $a \notin A$

Representation of Sets

(a) Roster (Tabular) Form
Elements are listed within curly braces.
Example:
A = {2, 4, 6, 8}
(b) Set-Builder Form
Elements are described using a property.
Example:
A = {x | x is an even natural number less than 10}
© . **Venn Diagrams**
Graphical representation of sets using closed curves.

Types of Sets

Empty (Null) Set (∅)
A set with no elements.
Example: A = {x | x is a natural number less than 0}
Singleton Set
A set containing exactly one element.
Example: A = {5}
Finite Set
A set with a finite number of elements.
Example: A = {1, 2, 3}
Infinite Set
A set with infinite elements.
Example: N = {1, 2, 3, ...}

- **Equivalent Sets**

Two sets having the same number of elements.

- **Subset ($\subseteq$)**

$A \subseteq B$ if every element of A is in B.

- **Proper Subset ($\subset$)**

$A \subset B$ if $A \subseteq B$ and $A \neq B$.

- **Universal Set (U)**

The set containing all elements under consideration.

Set Operations

Union:

$$A \cup B = \{x: x \ni A \; or \, x \in B\}$$

Intersection:

$$A \cap B = \{x: x \ni A \; and \, x \in B\}$$

Difference:

$$A - B = \{x: x \ni A \; and \, x \notin B\}$$

Complement:

$$A' = U - A$$

Example

Let $A = \{1, 2, 3\}, B = \{3, 4, 5\}$

$$A \cup B = \{1, 2, 3, 4, 5\}$$
$$A \cap B = \{3\}$$

Types of Relations

Reflexive:

$$(a,a) \ni R \; \forall a \ni A$$

Symmetric:

$$(a,b) \ni R \Rightarrow (b,a) \ni R$$

Transitive:

$$(a,b), (b,c) \ni R \Rightarrow (a,c) \ni R$$

Equivalence Relation:
Reflexive + Symmetric + Transitive
Partial Order Relation:
Reflexive + AntiSymmetric + Transitive
Poset
A set with a partial order relation is called a **Partially Ordered Set (Poset)**.

Problem

Let $A = \{1,2,3\}$and

$$R = \{(1,1),(2,2),(3,3),(1,2),(2,1)\}$$

Check whether $R$is an equivalence relation.

Solution

Reflexive:

$$(1,1), (2,2), (3,3) \ni R$$

Symmetric:

$$(1,2) \ni R \Rightarrow (2,1) \ni R$$

Transitive:

$$(1,2), (2,1) \Rightarrow (1,1) \ni R$$

$$\boxed{\text{R } \textit{is an equivalence relation}}$$

Problem

Define relation R on $\mathbb{Z}$ by

$$aRb \iff a - b \text{ is even}$$

Solution

Reflexive: $a - a = 0$ even ✓

Symmetric: $a - b$ even $\Rightarrow b - a$ even ✓

Transitive: ✓

$$\boxed{\text{Equivalence } \textit{relation}}$$

PROBLEMS ON PARTIAL ORDER RELATION

Let $A = \{1,2,3\}$ and

$$R = \{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}$$

Check whether R is a partial order relation.

Solution

- ✓ Reflexive
- ✓ Antisymmetric
- ✓ Transitive

$$\boxed{\text{Partial } \textit{order relation}}$$

HASSE DIAGRAM PROBLEM

Draw the **Hasse diagram** for the set

$$A = \{1,2,3,6\}$$

with relation “divides”.

Solution

Relations:

$$1|2,\ \ 1|3,\ \ 2|6,\ \ 3|6$$

Hasse Diagram (text form):

```
   6
  / \
 2   3
  \ /
   1
```

Problem

Draw Hasse diagram for

$$A = \{2,4,8\}$$

8
|
4
|
2

FUNCTIONS

Definition
A **function** $f: A \to B$assigns **exactly one element of B** to each element of A.
Types of Functions
One-to-One (Injective):
Different inputs → different outputs
Onto (Surjective):
Every element of B has a pre-image
Bijective:
Both one-to-one and onto

Example

Let $f(x) = 2x$,$A = \{1,2,3\}$

$$f(1) = 2,\ f(2) = 4,\ f(3) = 6$$

Different inputs give different outputs.So f is one – one.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$be defined by

$$f(x) = 3x + 5$$

Check whether $f$is bijective.

Solution

- ✓ One-to-one: linear function
- ✓ Onto: every real number has a preimage

$$\boxed{\text{Bijective } function}$$

FINDING INVERSE FUNCTION

Find the inverse of

$$f(x) = 2x + 1$$

Solution

$$y = 2x + 1 \Rightarrow x = \frac{y - 1}{2}$$

$$\boxed{f^{-1}(x) = \frac{x - 1}{2}}$$

COMPOSITE FUNCTION PROBLEM

Let

$$f(x) = x + 2 \ , g(x) = x^2$$

Find:

$$(f \circ g)(x)$$

$$(g \circ f)(x)$$

Solution

$$(f \circ g)(x) = f\big(g(x)\big) = x^2 + 2$$

$$(g \circ f)(x) = g(x + 2) = (x + 2(^2$$


Thank you