Discrete Mathematics

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UNIT-I

Mathematical logic

Propositions

- A proposition is a declarative sentence that is either true or false.
- Examples of propositions:

  The Moon is made of green cheese.

  The sky is blue

  $2 + 2 = 4$

  It is raining

Examples that are not propositions.

  Sit down!

  What time is it?

  $x + 1 = 2$

  $x + y = z$

Basic Logical Connectives

1. Negation ($\neg$ P): Reverses the truth value.

   Ex. P:"It is raining.“ $\neg$ P:"It is not raining.“

p	¬p
T	F
F	T

2. Conjunction (P∧Q): True if both P and Q are true

   Ex. " If P denotes “I am at home.” and Q denotes “It is raining.”
   then P ∧Q is “I am at home and it is raining.”

3. "Disjunction (P∨Q): False if both P and Q are false.

   Ex. "If P denotes “I am at home.” and Q denotes “It is raining.”
   then p ∨q denotes “I am at home or it is raining.”

4. "Implication (P→Q): True except when P is true and Q is false.

   Ex. "If it is raining, then the ground is wet.

Converse, Inverse, and Contrapositive of a Conditional Statement

Given Conditional Statement: "If it rains, then the ground is wet." ($P \rightarrow Q$)

Converse: "If the ground is wet, then it rains." ($Q \rightarrow P$)

Inverse: "If it does not rain, then the ground is not wet."
($\neg P \rightarrow \neg Q$)

Contrapositive: "If the ground is not wet, then it does not rain." ($\neg Q \rightarrow \neg P$)

- The contrapositive ($\neg Q \rightarrow \neg P$) is always logically equivalent to the original statement ($P \rightarrow Q$).

Logical Equivalences

Two propositions are e*quivalent* if they always have the same truth value.

Ex. ¬ **(P∨Q)≡ (¬ P∧¬ Q)**

Proof Using Truth Tables:

Since the columns for ¬(P ∨Q) and ¬P∧¬Q in the following table match, they are logically equivalent.

P	Q	P ∨Q	¬(P ∨Q)	¬P	¬Q	¬P∧¬Q
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Tautologies, Contradictions, and Contingencies

- A *tautology* is a proposition which is always true.

  Example: $p \vee \neg p$
- A *contradiction* is a proposition which is always false.

  Example: $p \wedge \neg p$
- A *contingency* is a proposition which is neither a tautology nor a contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

De Morgan’s La

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

p	q	$\neg p$	$\neg q$	$(p \vee q)$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Key Logical Equivalences

- Identity Laws: $p \wedge T \equiv p$ $\quad p \vee F \equiv p$
- Domination Laws: $p \vee T \equiv T$ $\quad p \wedge F \equiv F$
- Idempotent laws: $p \vee p \equiv p$ $\quad p \wedge p \equiv p$
- Double Negation Law: $\neg(\neg p) \equiv p$
- Negation Laws: $p \vee \neg p \equiv T$ $\quad p \wedge \neg p \equiv F$

Key Logical Equivalences

- Commutative Laws: , $p \vee q \equiv q \vee p$ $\quad p \wedge q \equiv q \wedge p$
- Associative Laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

  $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- Distributive Laws:

  $(p \vee (q \wedge r)) \equiv (p \vee q) \wedge (p \vee r)$

  $(p \wedge (q \vee r)) \equiv (p \wedge q) \vee (p \wedge r)$
- Absorption Laws: $p \vee (p \wedge q) \equiv p$

  $p \wedge (p \vee q) \equiv p$

Disjunctive Normal Form (DNF) and Conjunctive Normal Form (CNF)

- **Disjunctive Normal Form (DNF):**
  - A formula in DNF is a disjunction (OR) of conjunctions (ANDs) of literals.
  - Steps to Convert to DNF:
    - Apply De Morgan’s Laws if necessary.
    - Distribute disjunctions over conjunctions.
    - Ensure the formula is a disjunction of AND clauses.

    Example. $(\neg P \wedge Q) \vee (P \wedge R)$
- **Conjunctive Normal Form (CNF):**
  - A formula in CNF is a conjunction (AND) of disjunctions (ORs) of literals.
  - Steps to Convert to CNF:
    - Apply De Morgan’s Laws if necessary.
    - Distribute conjunctions over disjunctions.
    - Ensure the formula is an AND of OR clauses.

    Example. $(\neg P \vee Q) \wedge (P \vee \neg R)$

Principal Disjunctive Normal Form (PDNF) & Principal Conjunctive Normal Form (PCNF)

ion: $F(A, B, C) = (A \land \neg B) \lor (\neg A \land B \land C)$

Minterms): Identify the true cases:

F, $C = T) \rightarrow A \land \neg B \land C$

F, $C = F) \rightarrow A \land \neg B \land \neg C$

T, $C = T) \rightarrow \neg A \land B \land C$

ession: $(A \land \neg B \land C) \lor (A \land \neg B \land \neg C) \lor (\neg A \land B \land C)$

of Maxterms): Identify the false cases:

T, $C = T) \rightarrow \neg A \lor \neg B \lor \neg C$

T, $C = F) \rightarrow \neg A \lor \neg B \lor C$

T, $C = F) \rightarrow A \lor \neg B \lor C$

T, $C = T) \rightarrow A \lor B \lor \neg C$

T, $C = F) \rightarrow A \lor B \lor C$

on: $(\neg A \lor \neg B \lor \neg C) \land (\neg A \lor \neg B \lor C) \land (A \lor \neg B \lor C) \land (A \lor B \lor \neg C) \land (A \lor B \lor C)$

A	B	C	F(A, B, C)
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

Inference Laws in Propositional Logic

Inference laws are logical rules that allow us to derive conc from given premises. They are fundamental in mathematic reasoning and proofs.

Modus Ponens .$(P \rightarrow Q, P) \rightarrow Q$.
Example:
Premise 1: If it rains, the ground will be wet.
Premise 2: It is raining.
Conclusion: The ground is wet.

Modus Tollens .$(\neg Q, P \rightarrow Q) \rightarrow \neg P$
Example:
Premise 1: If it rains, the ground will be wet.
Premise 2: The ground is not wet.
Conclusion: It is not raining.

Hypothetical Syllogism (HS) (Transitivity of Implication)

$(P \rightarrow Q\ ,\ Q \rightarrow R) \rightarrow (P \rightarrow R)$.

Example:

1. Premise 1: If I study, I will pass the exam.
2. Premise 2: If I pass the exam, I will graduate.
3. Conclusion: If I study, I will graduate.

Disjunctive Syllogism (DS)

$(P \vee Q, \neg P) \rightarrow Q$

Example:

1. Premise 1: It is either sunny or raining.
2. Premise 2: It is not sunny.
3. Conclusion: It is raining.

More Inference Laws

Sum Rule $P \rightarrow P \vee Q$ or $Q \rightarrow P \vee Q$

Example:

Premise: It is raining.

Conclusion: It is raining or it is snowing.

Simplification $P \wedge Q \rightarrow P$ or $P \wedge Q \rightarrow Q$

Example:

Premise: It is raining and it is cold.

Conclusion: It is raining.

Conjunction $(P,Q) \rightarrow P \wedge Q$ is true.

Example:

1. Premise 1: It is raining.
2. Premise 2: It is cold.
3. Conclusion: It is raining and it is cold.

Resolution $(P \vee Q, \neg P \vee R) \rightarrow Q \vee R$

Example:

1. Premise 1: The train is late or I will take a taxi.
2. Premise 2: The train is not late or I will walk.
3. Conclusion: I will take a taxi or I will walk.

Derivations using Inference Laws

Premises: If the computer has power, it will turn on. The computer did not turn on. **Derivation:** Step 1: Apply Modus Tollens to Premises (1) and (2): Since and are true, we conclude (The computer does not have power). **Conclusion:** The computer does not have power (). This argument is valid because we derived the conclusion using Modus Tollens.	**Premises:** If a person studies, they will pass the exam. If a person passes the exam, they will graduate. The person studies. **Derivation:** Step 1: Apply Modus Ponens to Premises (1) and (3): Since and are true, we conclude (The person passes the exam). Step 2: Apply Modus Ponens again to Premises (2) and the result from Step 1: Since and are true, we conclude (The person graduates). **Conclusion:** The person graduates (). This argument is valid because we derived the conclusion using correct inference laws.	**Premises:** If it rains, then ground will be wet. If the ground is wet, then the match will be canceled. It is raining. **Derivation:** Step 1: Apply Modus Ponens to Premises (1) and (3): Since and are true, we conclude (The ground is wet). Step 2: Apply Modus Ponens again to Premises (2) and the result from Step 1: Since and are true, we conclude (The match will be canceled). **Conclusion:** The match will be canceled (). This argument is valid because we derived the conclusion using correct inference laws.

Understanding Logical Reasoning with Predicate Logic

Why Predicate Calculus?

•Expresses statements involving variables and quantifiers.
•Used in mathematics, computer science, and artificial intelligence.
•Helps in formal proofs, theorem proving, and reasoning.

Components of Predicate Logic

1. **Predicates:** Functions that return true or false based on input (e.g., P(x) = "x is prime").
2. **Variables:** Symbols representing objects (e.g., x, y, z).
3. **Quantifiers:** Universal ($\forall$) and existential ($\exists$).
4. **Logical Connectives:** $\wedge$ (AND), $\vee$ (OR), $\neg$ (NOT), $\rightarrow$ (IMPLIES).

Quantifiers

Universal Quantifier (∀)

- Symbol: $\forall x\, P(x)$
- Means "for all x, P(x) is true".
- Example: $\forall x\, (x > 0 \rightarrow x^2 > 0)$ (For all x greater than 0, x squared is also greater than 0).
- Example: ∀x (x is a mammal → x has a backbone) (All mammals have backbones).

Existential Quantifier (∃)

- Symbol: $\exists x\, P(x)$
- Means "there exists an x such that P(x) is true".
- Example: ∃x (x is an even prime) (There exists an x that is an even prime,

Negation of Quantified Statements

- $\neg(\forall x\ P(x)) \equiv \exists x\ \neg P(x)$
- $\neg(\exists x\ P(x)) \equiv \forall x\ \neg P(x)$
- Example: "Not all students passed" is equivalent to "Some students failed".
- Example: "Not all students passed" is equivalent to "Some students failed".
- Example: "Not every number is even" is equivalent to "There exists at least one number that is not even".

Applications of Predicate Logic

Example Problem

- Given: "All humans are mortal. Socrates is a human."
- Express in predicate logic:
  - H(x): x is human
  - M(x): x is mortal
  - $\forall x\ (H(x) \to M(x))$
  - H(Socrates)
  - Conclusion: M(Socrates) (Socrates is mortal).

Validity of an Inference

- Given premises:
  - $\forall x\,(I(x) \rightarrow R(x))$ (All integers are rational numbers).
  - $\exists x\,(I(x) \land P(x))$ (Some integers are powers of 2).
- Conclusion: $\exists x\,(R(x) \land P(x))$ (Some rational numbers are powers of 2).
- Since every integer is a rational number, any integer that is a power of 2 is also a rational number.
- Thus, the conclusion logically follows from the premises, making the inference valid.