UNIT 1: MATHEMATICAL LOGIC

PART – A : 1 MARK QUESTIONS WITH ANSWERS

1. Introduction & Statements

Q1. What is Mathematical Logic?
A: Mathematical Logic is the branch of mathematics that deals with reasoning using symbols, statements, and logical rules.

Q2. Define a statement.
A: A statement is a declarative sentence that is either **true or false**, but not both.

Q3. Give an example of a statement.
A: "2 + 3 = 5" is a statement.

Q4. Is "Open the door" a statement? Why?
A: No, because it is a command and has no truth value.

2. Logical Notation & Connectives

Q5. What are logical connectives?
A: Logical connectives are symbols used to connect statements to form compound statements.

Q6. List any four logical connectives.
A:

- Negation (¬)
- Conjunction (∧)
- Disjunction (∨)
- Implication (→)

Q7. What does ¬p mean?
A: ¬p means "not p".

Q8. Write the logical notation for "p and q".
A: $p \wedge q$

Q9. Write the logical notation for "p implies q".
A: $p \rightarrow q$

3. Truth Tables

Q10. What is a truth table?
A: A truth table shows all possible truth values of a compound statement.

Q11. How many rows are there in a truth table with 3 variables?
A: $2^3 = 8$rows.

4. Well-Formed Formulas (WFF)

Q12. Define a well-formed formula.
A: A well-formed formula is a logically correct expression constructed using rules of syntax.

Q13. Is (p ∧ q →) a WFF?
A: No, because it is syntactically incorrect.

5. Tautology, Contradiction, Contingency

Q14. Define tautology.
A: A tautology is a compound statement that is **always true**.

Q15. Define contradiction.
A: A contradiction is a compound statement that is **always false**.

Q16. Define contingency.
A: A contingency is a statement that is **sometimes true and sometimes false**.

6. Logical Equivalence

Q17. When are two statements logically equivalent?
A: When they have the same truth values for all possible cases.

Q18. Write the equivalence of implication.
A:

$$p \to q \equiv \neg p \lor q$$

7. Normal Forms

Q19. What is Conjunctive Normal Form (CNF)?
A: CNF is a conjunction of disjunctions of literals.

Q20. What is Disjunctive Normal Form (DNF)?
A: DNF is a disjunction of conjunctions of literals.

8. Predicate Calculus

Q21. What is a predicate?
A: A predicate is a statement containing variables whose truth depends on the variable values.

Q22. What does ∀ mean?
A: ∀ means "for all" (universal quantifier).

Q23. What does ∃ mean?
A: ∃ means "there exists" (existential quantifier).

PART – B : 5 MARK QUESTIONS WITH DETAILED ANSWERS

Q1. Explain logical connectives with truth tables.

Answer:

Logical connectives are symbols used to combine simple statements.

1. Negation (¬p)

p	**¬p**
T	F
F	T

2. Conjunction (p ∧ q)

p	**q**	**p ∧ q**
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction (p ∨ q)

p	**q**	**p ∨ q**
T	T	T
T	F	T
F	T	T
F	F	F

4. Implication (p → q)

p	**q**	**p → q**
T	T	T
T	F	F
F	T	T

p	q	p → q
F	F	T

Q2. Define tautology, contradiction, and contingency with examples.

Tautology

A statement that is always true.

Example: $p \lor \neg p$

p	¬p	p ∨ ¬p
T	F	T
F	T	T

Prove the following Tautologies

1. $p \lor \neg p$
2. $(p \land q) \to p$
3. $p \to p$
4. $(p \to q) \lor (q \to p)$
5. $(p \to q) \equiv (\neg p \lor q)$
6. $(p \land (p \to q)) \to q$
7. $\neg(p \land \neg p)$
8. (p→q)→(¬q→¬p)

Contradiction

A statement that is always false.

Example: $p \land \neg p$
Prove the following CONTRADICTIONS

1. $p \land \neg p$
2. $\neg(p \lor \neg p)$
3. $(p \to q) \land p \land \neg q$
4. $\neg(p \to p)$

Contingency

A statement that is neither always true nor always false.

Example: $p \wedge q$

LOGICAL EQUIVALENCE LAWS

Equivalence Laws

Law Equivalence

Identity $p \wedge T \equiv p$

Domination $p \vee T \equiv T$

Idempotent $p \vee p \equiv p$

Double Negation $\neg(\neg p) \equiv p$

Commutative $p \wedge q \equiv q \wedge p$

Associative $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

Distributive $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

De Morgan $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Implication $p \rightarrow q \equiv \neg p \vee q$

Biconditional $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Q3. Explain logical equivalence and verify using truth table.

Answer:

Two statements are logically equivalent if they have identical truth values.

Verify:

$$p \rightarrow q \equiv \neg p \vee q$$

p	**q**	**p → q**	**¬p**	**¬p ∨ q**
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Since columns match, both are logically equivalent.

Q.Prove using laws $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Solution: Given $\neg(p \lor q)$
Applying **De Morgan's Law**:$= \neg p \land \neg q$
Hence proved.

Q.Show that $(p \to q) \land p \equiv p \land q$

Solution: Given $(p \to q) \land p$
Replace implication: $(\neg p \lor q) \land p$
Distribute: $(p \land \neg p) \lor (p \land q)$
Since $p \land \neg p = F{:} = p \land q$
Q4. Convert a statement into CNF and DNF.

Given: $(p \to q) \land r$
Step 1: Remove implication $(\neg p \lor q) \land r$
CNF: $(\neg p \lor q) \land r$
DNF: $(\neg p \land r) \lor (q \land r)$
Q5. Explain Theory of Inference (Statement Calculus).

Answer:

Inference theory deals with deriving conclusions from given premises.

Rules of Inference:Standard Inference Laws

No	Law	Form
1	Modus Ponens	$p \to q, p \Rightarrow q$
2	Modus Tollens	$p \to q, \neg q \Rightarrow \neg p$
3	Hypothetical Syllogism	$p \to q, q \to r \Rightarrow p \to r$
4	Disjunctive Syllogism	$p \lor q, \neg p \Rightarrow q$
5	Addition	$p \Rightarrow p \lor q$
6	Simplification	$p \land q \Rightarrow p$
7	Conjunction	$p, q \Rightarrow p \land q$
8	Resolution	$p \lor q, \neg p \lor r \Rightarrow q \lor r$
9	Constructive Dilemma	$(p \to q) \land (r \to s), p \lor r \Rightarrow q \lor s$
10	Destructive Dilemma	$(p \to q) \land (r \to s), \neg q \lor \neg s \Rightarrow \neg p \lor \neg r$

Q. Given Premises are: $p \to q$, $q \to r$, $r \to s$, p and Conclusion: s.

Solution.

Step	Statement	Reason
1	$p \to q$	Premise
2	$q \to r$	Premise
3	$r \to s$	Premise

Step	Statement	Reason
4	p	Premise
5	q	Modus Ponens (1,4)
6	r	Modus Ponens (2,5)
7	s	Modus Ponens (3,6)

Q. Given Premises: p ∨ q, p → r, q → s, ¬r and Conclusion: s

Solution.

Step	Statement	Reason
1	p ∨ q	Premise
2	p → r	Premise
3	q → s	Premise
4	¬r	Premise
5	¬p	Modus Tollens (2,4)
6	q	Disjunctive Syllogism (1,5)
7	s	Modus Ponens (3,6)

Q. Given Premises: p → q, r → s, p ∨ r, ¬q, Conclusion: s

Solution.

Step	Statement	Reason
1	p → q	Premise
2	r → s	Premise
3	p ∨ r	Premise
4	¬q	Premise
5	¬p	Modus Tollens (1,4)
6	r	Disjunctive Syllogism (3,5)
7	s	Modus Ponens (2,6)

Q. Given Premises: p → q, q → r, ¬r, p ∨ s, Conclusion: s

Solution.

Step	Statement	Reason
1	p → q	Premise
2	q → r	Premise
3	¬r	Premise
4	p ∨ s	Premise
5	¬q	Modus Tollens (2,3)
6	¬p	Modus Tollens (1,5)
7	s	Disjunctive Syllogism (4,6)

Q. Given Premises: p → q, r → s, ¬q ∨ ¬s, p, Conclusion: ¬s

Solution.

Step	Statement	Inference Law
1	$p \rightarrow q$	Premise
2	$r \rightarrow s$	Premise
3	$\neg q \vee \neg s$	Premise
4	p	Premise
5	q	Modus Ponens (1,4)
6	$\neg s$	Disjunctive Syllogism (3,5)

Q. Given Premises: $p \rightarrow q$, $q \rightarrow r$, $r \rightarrow s$, $\neg s$, $p \vee t$, Conclusion: t

Solution.

Step	Statement	Inference Law
1	$p \rightarrow q$	Premise
2	$q \rightarrow r$	Premise
3	$r \rightarrow s$	Premise
4	$\neg s$	Premise
5	$\neg r$	Modus Tollens (3,4)
6	$\neg q$	Modus Tollens (2,5)
7	$\neg p$	Modus Tollens (1,6)
8	t	Disjunctive Syllogism (5,7)

Q6. Explain Predicate Calculus with quantifiers.

Answer:

Predicate calculus extends statement calculus by including **variables and quantifiers**.

Example Predicate:

P(x): x is even

Universal Quantifier (∀):

$$\forall x\, P(x)$$

Means P(x) is true for all x.

Existential Quantifier (∃):

$$\exists x\, P(x)$$

Means P(x) is true for at least one x.

Q7. Explain inference theory of Predicate Calculus.

Answer:

Inference rules in predicate calculus include:

1. **Universal Instantiation**
   From ∀x P(x), infer P(a)
2. **Existential Instantiation**
   From ∃x P(x), infer P(c)
3. **Universal Generalization**
   From P(x), infer ∀x P(x)

Q. Given **Premises:**

1. ∀x (P(x) → Q(x))
2. ∀x (Q(x) → R(x))
3. P(a)

Conclusion: R(a)

Solution.

Step	**Statement**	**Inference Law**
1	∀x (P(x) → Q(x))	Premise
2	∀x (Q(x) → R(x))	Premise
3	P(a)	Premise
4	P(a) → Q(a)	Universal Instantiation (1)
5	Q(a)	Modus Ponens (3,4)
6	Q(a) → R(a)	Universal Instantiation (2)
7	R(a)	Modus Ponens (5,6)

Q. **Premises:**

1. ∀x (P(x) → Q(x))
2. ∃x P(x)

Conclusion: ∃x Q(x)

Solution.

Step	**Statement**	**Inference Law**
1	∀x (P(x) → Q(x))	Premise
2	∃x P(x)	Premise
3	P(c)	Existential Instantiation (2)
4	P(c) → Q(c)	Universal Instantiation (1)
5	Q(c)	Modus Ponens (3,4)
6	∃x Q(x)	Existential Generalization (5)

Q. Premises:

1. $\forall x\ (P(x) \rightarrow Q(x))$
2. $\exists x\ (P(x) \land R(x))$

Conclusion:

$\exists x\ Q(x)$

Solution.

Step	Statement	Inference Law
1	$\forall x\ (P(x) \rightarrow Q(x))$	Premise
2	$\exists x\ (P(x) \land R(x))$	Premise
3	$P(c) \land R(c)$	Existential Instantiation (2)
4	$P(c)$	Simplification
5	$P(c) \rightarrow Q(c)$	Universal Instantiation (1)
6	$Q(c)$	Modus Ponens (4,5)
7	$\exists x\ Q(x)$	Existential Generalization (6)

Q. Premises:

1. $\forall x\ (P(x) \rightarrow Q(x))$
2. $\exists x\ \neg Q(x)$

Conclusion:

$\exists x\ \neg P(x)$

Solution.

Step	Statement	Inference Law
1	$\forall x\ (P(x) \rightarrow Q(x))$	Premise
2	$\exists x\ \neg Q(x)$	Premise
3	$\neg Q(c)$	Existential Instantiation (2)
4	$P(c) \rightarrow Q(c)$	Universal Instantiation (1)
5	$\neg P(c)$	Modus Tollens (4,3)
6	$\exists x\ \neg P(x)$	Existential Generalization (5)

Q. Premises:

1. $\forall x\ (P(x) \rightarrow Q(x))$
2. $\forall x\ (Q(x) \rightarrow R(x))$
3. $\forall x\ (R(x) \rightarrow S(x))$
4. $\exists x\ P(x)$

Conclusion:

$\exists x\, S(x)$

Solution.

Step	Statement	Inference Law
1	$\forall x\, (P(x) \rightarrow Q(x))$	Premise
2	$\forall x\, (Q(x) \rightarrow R(x))$	Premise
3	$\forall x\, (R(x) \rightarrow S(x))$	Premise
4	$\exists x\, P(x)$	Premise
5	$P(a)$	Existential Instantiation (4)
6	$P(a) \rightarrow Q(a)$	Universal Instantiation (1)
7	$Q(a)$	Modus Ponens
8	$Q(a) \rightarrow R(a)$	Universal Instantiation (2)
9	$R(a)$	Modus Ponens
10	$R(a) \rightarrow S(a)$	Universal Instantiation (3)
11	$S(a)$	Modus Ponens
12	$\exists x\, S(x)$	Existential Generalization